Northwestern

Math 290-2 Midterm 2
Winter Quarter 2019
March 4, 2019

Last name: ______________________ Email address: ______________________

First name: ______________________ NetID: ______________________

Instructions

- Mark your instructor's name.

  _____ Cañez

  _____ Newstead

  _____ Norton

- This examination consists of 11 pages, not including this cover page. Verify that your copy of this examination contains all 11 pages. If your examination is missing any pages, then obtain a new copy of the examination immediately.
- This examination consists of 6 questions for a total of 100 points.
- You have one hour to complete this examination.
- Do not use books, notes, calculators, computers, tablets, or phones.
- Write legibly and only inside of the boxed region on each page.
- Cross out any work that you do not wish to have scored.
- Show all of your work. Unsupported answers may not earn credit.

1. Determine whether each of the following statements is **TRUE** or **FALSE**. Justify your answer. (This question has **four** parts. Remember: A statement is "true" if it is always true. If not, it is "false.")

   (a) (5 points) The surface in $\mathbb{R}^3$ defined by $x^2 + 2yz = 1$ is a hyperboloid of one sheet.

   Answer:

   (b) (5 points) The surface in $\mathbb{R}^3$ defined in cylindrical coordinates by $z^2 = 1 + r^2 \cos(2\theta)$ is an ellipsoid. (Recall: $\cos(2\theta) = \cos^2\theta - \sin^2\theta$.)

   Answer:

(c) (5 points) The function

$$f(x,y) = \begin{cases} e^{xy} + \frac{x^5}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$$

is continuous at $(0,0)$.

Answer:

(d) (5 points) Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is differentiable at $(3,0)$ and $\nabla f(3,0) = (1,2)$. If $\vec{x}(t) = (3\cos t, 4\sin t)$, then $D(f \circ \vec{x})(0) = 8$.

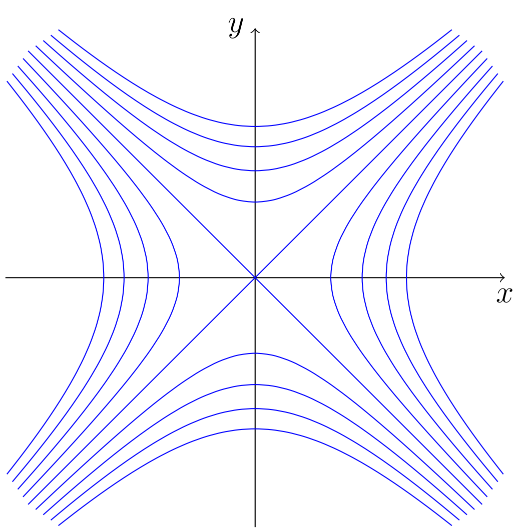
Answer:

2. Determine whether each of the following statements is **ALWAYS** true, **SOMETIMES** true, or **NEVER** true. Justify your answer. (This question has **four** parts.)

(a) (5 points) Suppose $f : \mathbb{R}^2 \to \mathbb{R}^2$ is differentiable. Then $Df(0,0) = Df(0,0)^T$.

Answer:

(b) (5 points) The following diagram represents the level curves of the surface $z = \begin{bmatrix} x & y \end{bmatrix} A \begin{bmatrix} x \\ y \end{bmatrix}$, where A is a 2×2 symmetric matrix with positive determinant.



Answer:

(c) (5 points) Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is differentiable at $\vec{a} \in \mathbb{R}^2$, and suppose there exist linearly independent unit vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^2$ such that $D_{\vec{u}} f(\vec{a}) = D_{\vec{v}} f(\vec{a}) = 0$. If $\vec{w}$ is a unit vector in $\mathbb{R}^2$, then $D_{\vec{w}} f(\vec{a}) = 0$.

Answer:

(d) (5 points) Let (a, b, c) be a point in $\mathbb{R}^3$, and let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined by $f(x, y, z) = x^3 + e^y + xz^3$. Then the following matrix is invertible.

$$\begin{bmatrix} f_{xx}(a,b,c) & f_{xy}(a,b,c) & f_{xz}(a,b,c) \\ f_{yx}(a,b,c) & f_{yy}(a,b,c) & f_{yz}(a,b,c) \\ f_{zx}(a,b,c) & f_{zy}(a,b,c) & f_{zz}(a,b,c) \end{bmatrix}$$

Answer:

3. Suppose the depth of a pond at the point (x, y) is given by the function $f(x, y) = 50 - 4x^2y^2 - x^2 - 2y^2$. A child is standing at the point $(-3, 1)$ and can only just touch the bottom.

   (a) (6 points) In what direction could the child walk so that the depth decreases most rapidly (i.e., so they can easily touch the bottom)? Your answer does *not* need to be expressed as a unit vector

   (b) (4 points) In what direction could the child walk so that the depth remains constant? Justify your answer. Your answer does *not* need to be expressed as a unit vector.

   (c) (8 points) If the child swims directly toward the center of the pond (i.e., toward $(0, 0)$), at what rate is the depth changing at the moment when they start swimming?

4. (10 points) Let $f(x, y) = xye^y$. Find all points (a, b) such that the tangent plane to the graph of f at $(a, b, f(a, b))$ is parallel to the plane $8y - 2z = 10$.

5. (16 points) Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $f(x, y, z) = (x^2 + z, e^x)$, and let $x(r, \theta, z), y(r, \theta, z)$ and $z(r, \theta, z)$ be the expressions for the Cartesian coordinates (x, y, z) in terms of the cylindrical coordinates (r, θ, z). Find the derivative (i.e. Jacobian matrix) $Dg(r, \theta, z)$ of the function $g(r, \theta, z) = f(x(r, \theta, z), y(r, \theta, z), z(r, \theta, z))$ at the point with Cartesian coordinates $(x, y, z) = (0, 1, 2)$.

6. Define planes P_1, P_2 and P_3 as follows:

$$\begin{cases} P_1: & x & & - & z & = & 0 \\ P_2: & & y & + & 2z & = & 1 \\ P_3: & x & + \; y & + & z & = & -2 \end{cases}$$

(a) (8 points) Verify that the line of intersection of the planes P_2 and P_3 does not intersect the plane P_1.

(b) (8 points) Find the distance from P_1 to the line of intersection of P_2 and P_3.

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If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.

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