Northwestern University **Name:**

Math 290-3: Midterm 2

Spring Quarter 2015

Thursday, May 21, 2015

Put a check mark next to your section:

Davis (10am)		Canez	
Peterson		Davis (12pm)	

Question	Possible points	Score
1	20	
2	20	
3	15	
4	15	
5	15	
6	15	
TOTAL	100	

Instructions:

- Read each problem carefully.
- Write legibly.
- Show all your work on these sheets.
- This exam has 10 pages, and 6 questions. Please make sure that all pages are included.
- You may not use books, notes or calculators.
- You have one hour to complete this exam.

Good luck!

1. Determine whether each of the following statements is **TRUE** or **FALSE**. Justify your answer. (This problem has **four** parts.)

   (a) Let $\mathbf{F}(x, y) = (0, x)$ and let C be the ellipse $x^2 + y^2/4 = 1$ oriented counter-clockwise. Then $\int_C \mathbf{F} \cdot d\mathbf{s} > 0$.

Answer:

   (b) Let $\mathbf{F}(x, y) = (2xy + \sin y, x^2 + x\cos y)$ and let C be the curve with parametric equations $\mathbf{x}(t) = (\sin(\pi t), t^2)$ for $-1 \leq t \leq 1$. Then $\int_C \mathbf{F} \cdot d\mathbf{s} > 0$.

Answer:

(c) Let D be the region in $\mathbb{R}^2$ obtained by removing the origin. If $\mathbf{F}$ is a C^1 vector field on D such that $\operatorname{curl}\mathbf{F} = \mathbf{0}$ at each point of D, then $\oint_C \mathbf{F}\cdot d\mathbf{s} = 0$ for every closed curve C in D.

Answer:

(d) Let $\mathbf{F}(x, y) = (P(x, y), Q(x, y))$, let D be the rectangle $[0, 1] \times [0, 2]$ and let C be the curve consisting of the line segment from $(0, 0)$ to $(1, 0)$, followed by the line segment from $(1, 0)$ to $(1, 2)$, followed by the line segment from $(1, 2)$ to $(0, 2)$. Assume that $\mathbf{F}$ has continuous partial derivatives in D. Then

$$\int_C \mathbf{F}\cdot d\mathbf{s} = \int_0^1 \int_0^2 \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dy\,dx + \int_0^2 Q(0, t)\,dt.$$

Answer:

2. Determine whether each of the following statements is **ALWAYS** true, **SOMETIMES** true, or **NEVER** true. Justify your answer. (This problem has **four** parts.)

   (a) For numbers $a < b$, let $\mathbf{x}(t) = (x(t), y(t))$ with $a \leq t \leq b$ be parametric equations for a smooth curve such that $\mathbf{x}(t)$ lies on the unit circle for all t. Then

   $$\int_a^b \|\mathbf{x}'(t)\|\, dt = 2\pi.$$

Answer:

   (b) Let C be the portion of the curve defined by $1 = ye^{-x^2}$ which starts at $(-1, e)$ and ends at $(1, e)$. For a real number k,

   $$\int_C -2xye^{-x^2}\,dx + (e^{-x^2} + k^2)dy > 0.$$

Answer:

(c) For a C^1 vector field $\mathbf{F}$ on $\mathbb{R}^2$ such that $\text{div}(\mathbf{F}) = 0$ everywhere, $(\text{curl}\,\mathbf{F})(p) \neq \mathbf{0}$ for every point p in $\mathbb{R}^2$.

Answer:

(d) For a **nonzero** C^2 vector field $\mathbf{F}$ on $\mathbb{R}^3$, we have $\text{curl}\,\mathbf{F} = \nabla(\text{div}\,\mathbf{F})$.

Answer:

3. Determine the value of the scalar line integral

$$\int_C (2xy - yz)\, ds$$

where C is the intersection of the cylinder $y^2 + z^2 = 1$ and the plane $z = x$.

4. Compute the vector line integral

$$\int_C (y + \sin y + e^{x^4})\,dx + (y + (x-1)\cos y)\,dy$$

where C is the left half of the circle $(x-1)^2 + y^2 = 1$ oriented clockwise.

5. (This question has **two** parts.) Let $\mathbf{F}$ be the vector field on $\mathbb{R}^2$ defined by

$$\mathbf{F}(x,y) = (ye^y + y^2 - y\pi\sin(xy\pi))\mathbf{i} + (xe^y + xye^y + 2yx - x\pi\sin(xy\pi))\mathbf{j}.$$

(a) Show that $\mathbf{F}$ is conservative on $\mathbb{R}^2$.

(b) Find the value of the vector line integral

$$\int_C (y + ye^y + y^2 - y\pi \sin(xy\pi))\, dx + (-x + xe^y + xye^y + 2yx - x\pi \sin(xy\pi))\, dy$$

where C is the piece of the parabola $x = y^2 - 1$ which starts at $(0, -1)$ and ends at $(0, 1)$.

6. Compute the vector line integral

$$\int_C (z + y\sin^2(z+1))\,dx - x\cos^2(z+1)\,dy + z^{100}e^{x\cos y}\,dz$$

where C is the square $[0,1] \times [0,1]$ in the xy-plane oriented counterclockwise when viewed from the positive z-direction.