Math 306: Midterm 1

Northwestern University, Winter 2019

Name: ______________________________

1. (10 points) List the following things. No justification is needed, just list them.

(a) The compositions of 10 into an even number of even parts.

(b) The parenthetical expressions defining the 3-rd Catalan number, or, if you prefer, the paths in a 3×3 grid which define the 3-rd Catalan number.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Do either (a) or (b). (You can do them both if you'd like for 2 points extra credit.)

(a) Let $n \geq 2$. Show that if we select $n+1$ integers from the set $[2n]$, there will be two among them so that one is a multiple of the other.

(b) Let $n \geq 3$. Show that any convex n-gon can be split up into $n-2$ triangles by drawing line segments which connect vertices. (A *convex* polygon is one where these line segments lie within the polygon.)

3. (10 points) Let $n \geq 4$. Determine the number of subsets of $[n]$ which contain at least one of 1 or 2, and *at the same time* exactly one of 3 or 4.

4. Justify the following identity by interpreting both sides as counting the same thing.

$$n\binom{2n-1}{n-1} = \sum_{k=1}^{n} k\binom{n}{k}^2$$

Hint: Think about a construction involving picking a committee from a group of $2n$ people. Thinking about the identity $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$ first might help.

5. (10 points) Let a_n denote the number of compositions of n into odd parts. (That is, parts each of which are odd, *not* necessarily an odd number of parts overall.) Compute a_1, a_2, a_3, a_4, a_5 and determine, with justification, a recursive identity for a_n in terms of some (which ones to use is up to you) a_k for smaller k.