Math 306: Midterm 2 Northwestern University, Winter 2019

Name: _____

1. (10 points) List out all the partitions (set partitions in the first case and integer partitions in the second) which are counted by the following numbers.

(a) S(4, 2)(b) p(6)

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Show that the number of integer partitions of n in which the two largest parts are equal is p(n) - p(n-1), where p(k) denotes the number of all partitions of k. You cannot take any fact derived in the book, or in class, or elsewhere for granted, and must justify any result you make use of. For instance, you cannot use without justification any fact about the number of partitions of n where all parts are of at least a certain size.

3. (10 points) Compute the unsigned Stirling number c(9,3). You should express your answer as a concrete sum, but you do not have to simplify factorials, products, nor quotients, so that your answer does not have to be a single explicit number. Hint: What types of cycles do you need to describe the permutations counted by c(9,3)?

4. (10 points) Find a formula for the number of set partitions of [3m] in which no multiple of 3 is in a singleton block, meaning the partition does not contain $\{3\}$, nor $\{6\}$, nor $\{9\}$, etc. Hint: First find a summation formula for the number of partitions which *do* contain such a singleton block. You will need to express your answer in terms of Bell numbers B(k), which if you recall describe the number of all set partitions of [k]. 5. Say we have *n* people arranged in a line, which we split into at most two groups by placing a division point somewhere in the line; so, we never mix up the order of the people, and if one person ends up in the first group then all people before that person in the line also go into the first group. We allow for the possibility that one group is empty, which is why we get at most two groups. To each person in the first group we give either a red or blue hat, and to each person in the second group we give either a green, white, or black hat.

(a) (5 points) Explain why the ordinary generating function for the number of ways of carrying out this process is

$$\frac{1}{(1-2x)(1-3x)}$$

(b) (5 points) Find an explicit formula, which does not use a summation, for the number of ways of carrying out this process. (You should be able to do this part even if you can't figure out the first part.)