Northwestern University

MATH 320-1 Midterm 1
Fall Quarter 2023
October 17, 2023

Last name: _____ Email address: _____

First name: _____ NetID: _____

This is just the cover page Crowdmark forces me to put on the front. The only thing necessary to fill out is your name. No email address nor NetID is actually needed.

There are five problems. Be clear in your work about what is scratch work and what is what you actually want graded. There are three extra pages towards the back; if you use these pages for work you want graded, be sure to indicate to which problem the work corresponds.

1. Give an example of each of the following. You do not have to justify your answer.

   (a) (3 points) A subset of $\mathbb{R}$ which has a supremum but not an infimum.

   (b) (4 points) An unbounded sequence in $\mathbb{R}$ with at least two convergent subsequences.

   (c) (3 points) A Cauchy sequence all of whose terms are irrational.

2. (10 points) Show that the supremum of the following set is 4.

$$A = \left\{ \frac{4n^2 - 6n}{n^2 - n + 1} \;\middle|\; n \in \mathbb{N} \right\}$$

3. Suppose the sequence (x_n) converges to $1/2$. Show, using the definition of convergence, that the sequence $(3/x_n^2)$ converges to $12 = 3/(1/2)^2$.

4. (10 points) As a consequence of a problem on the homework, the sequence (x_n) defined by $x_n = \frac{\pi^n}{n!}$ converges to 0. Using the fact that

$$x_{n+1} = \frac{\pi}{n+1} x_n \text{ for } n \geq 1,$$

give an alternative proof that $x_n \to 0$ which does not directly use the definition of convergence.

Hint: Which of b_4 and b_5 is larger? Which of b_5 and b_6 is larger? What about b_6 and b_7?

5. (10 points) Suppose (x_n) and (z_n) are convergent sequences and that (y_n) is a sequence such that

$$x_n \leq y_n \leq z_n \text{ for } n \geq 10.$$

Show that (y_n) has a convergent subsequence.

Careful: We are not assuming that (x_n) and (z_n) converge to the same thing, so no squeeze theorem applies.

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