061 - Midterm 2 - Practice Problems

14 May 2011

1. Solve the recurrence relation

$$a_n = 5a_{n-1} - 6a_{n-2}$$

subject to the initial conditions $a_0 = 5$ and $a_1 = 4$.

2. Let a_n be the number of strings of length n on the set $X = \{0, 1, 2\}$ with no consecutive 0s. Find a linear homogeneous recurrence relation of order 2 satisfied by a_n .

3. Let K be the graph



How many paths of length 3 from v_1 to v_2 are there?

4. Prove that the graph G



does not have a Hamiltonian cycle.

5. Prove that there is no simple graph with 5 vertices such that the degree of every vertex is 3.

6. Which complete bipartite graphs $K_{m,n}$ with m > 0 and n > 0 have Euler cycles?

7. Consider the weighted graph G



Run Dijkstra's algorithm to find the length of the shortest path from a to z. Draw the state of the graph when the algorithm finishes together with all labels permanent or not on all vertices.

8. Let G be a simple graph with 11 vertices. Show that either G or its complement \overline{G} is not planar.