

31/B - Practice Final

November 28, 2011

1. (20 points) Calculate $g(1)$ and $g'(1)$, where $g(x)$ is the inverse of $f(x) = x + \cos x$.

2. (20 points) Evaluate

$$\int \frac{dx}{x^2 \sqrt{5-x^2}}$$

using trigonometric substitution.

3. (20 points) Evaluate the integral

$$\int \frac{x^4 + 1}{x(x+1)^2} dx.$$

4. (20 points) Use the error bound for Simpson's Rule to find an integer N for which $error(S_N) \leq 10^{-15}$ in the integral

$$\int_1^5 \frac{dx}{x}.$$

5. (20 points) Calculate the arc length of $y = \frac{1}{4}x^2 - \frac{1}{2} \ln x$ over the interval $[1, 2e]$.

6. (20 points) Find the limit

$$\lim_{n \rightarrow \infty} \frac{(\ln n)^2}{n}.$$

7. (20 points) Use the error bound to find a value of n for which

$$|e^{-0.1} - T_n(-0.1)| \leq 10^{-6},$$

where T_n is the n th Taylor polynomial for $f(x) = e^x$ with center 0.

8. (20 points) For which real numbers a does

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^a}$$

converge?

9. (20 points) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{x^n}{n3^n}.$$

10. (20 points) Find the terms through degree 5 of the Taylor series $T(x)$ centered at $c = 0$ of $f(x) = e^x \tan^{-1} x$.