548 - Spring 2018 - HW1

January 19, 2018

1. Prove that $X \vee Y$ is the coproduct in $\mathcal{T}_*$.

2. Suppose that $F : \mathcal{C} \rightleftarrows \mathcal{D} : G$ are adjoint functors (with F being the left adjoint). Show that F preserves any colimits that exist in $\mathcal{C}$ and that G preserves any limits that exist in $\mathcal{D}$.

3. Use Exercise 2 to show that the product of two pointed spaces (X, x) and (Y, y) in $\mathcal{T}$ is given by $(X \times Y, (x, y))$.

4. Prove that $X \wedge S^0$ and X are homeomorphic as pointed spaces.

5. Let (X, x) be a based space and let X_+ denote $X \coprod \{+\}$ pointed by $+$. Show that there is a homotopy equivalence between $S^1 \wedge X_+$ and $(S^1 \wedge (X, x)) \vee S^1$.

6. Prove that $\pi_1 S^1 \cong \mathbb{Z}$.

7. Prove that $\pi_n S^1 = 0$ for $n > 1$ using a lifting property, as in Proposition 1.33 of Hatcher.

8. Prove that if $f : S^m \to S^n$ is nullhomotopic, then C_f is homotopy equivalent to $S^n \vee S^{m+1}$.