547 - Fall 2019 - HW2

Due 13 September 2019

1. Let $f, g, h : S^1 \to X$ be continuous pointed functions. Write down an explicit homotopy between $(f \cdot g) \cdot h$ and $f \cdot (g \cdot h)$.

2. Suppose that $a : I^1 \to X$ is a path from $a(0)$ to $a(1)$. Prove that "conjugation by a",

$$f \mapsto a^{-1} \cdot f \cdot a,$$

gives a well defined isomorphism $\pi_1(X, a(0)) \cong \pi_1(X, a(1))$.

3. Let $f, g : X \to Y$ be pointed maps. If f is homotopic to g, then $f_* = g_* : \pi_1(X, x) \to \pi_1(Y, y)$.

4. Prove that π_1 induces a functor $\mathrm{Ho}(\mathcal{T}_*) \to \mathrm{Groups}$.

5. Describe pushouts and pullbacks in the category Ab of abelian groups.

6. Describe pushouts and pullbacks in the category CAlg of commutative $\mathbb{Z}$-algebras (i.e., commuting rings).

7. Does the forgetful functor Ab $\leftarrow$ CAlg preserve pullbacks? What about pushouts?

8. Prove that $\pi_1(\mathbb{P}^2(\mathbb{C})) = 0$. You can do this using van Kampen's theorem.

9. Do Hatcher, Exercise 1.1.16.

10. Do Hatcher, Exercise 1.1.18.