

547 - Fall 2019 - HW3

Due 20 September 2019

1. Prove that if $E \xrightarrow{p} B$ and $F \xrightarrow{q} B$ are covering spaces, then so is $E \amalg F \xrightarrow{p \amalg q} B$.
2. Show that if $E \xrightarrow{p} B$ is a covering space and $X \rightarrow B$ is a map with X connected and locally path connected, then $E \times_B X \rightarrow X$ is a covering space.
3. Let G be a discrete group acting freely and continuously on a topological space X . Assume that X is locally path connected. What condition on the action is needed to guarantee that $X \rightarrow X/G$ is a covering space? What are the fibers in this case?
4. Suppose that G is a discrete group acting freely on a simply connected space X . Compute $\pi_1(X/G)$.
5. Suppose that G and H are non-zero groups. Find the center of $G * H$.
6. Let X be the complement of finitely many points in \mathbb{R}^n for $n \geq 3$. Compute $\pi_1(X)$.
7. Let X be the complement of finitely many points in \mathbb{R}^2 . Compute $\pi_1(X)$.
8. Let X be a path-connected space and consider a map $S^n \xrightarrow{f} X$ where $n \geq 2$. Compute $\pi_1(C_f)$.
9. Prove that $\pi_1(\mathbb{R}^2 - \mathbb{Q}^2)$ is uncountable.
10. Do Hatcher, Exercise 1.2.7.