Welcome to "Large deviations through examples"!

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The goal of the course is to give a first taste of large deviations by focusing on very few results and techniques and illustrating them by examples. Problem sessions will offer an opportunity to apply the material discussed in lectures and expand it in various directions.

Level and prerequisites: The course is intended for advanced undergraduate and beginning graduate students. Large deviations are about asymptotic behavior of probability measures, hence, we expect that students

(i) have taken a course in probability and familiar with basic combinatorics, probability distributions, random variables, expectations, moment generation functions, Markov and related inequalities, laws of large numbers, and the central limit theorem;

(ii) have taken a course in analysis and are very comfortable with such notions as open, closed, compact sets, rigorous definitions of infimum/supremum, limits, continuity, convexity;

(iii) are familiar with basic definition and properties of probability measures and expectation as an integral with respect to a probability measure.

It would be useful (but not really necessary) to know the definition of weak convergence of probability measures and Portmanteau Theorem (see, for example, the first two pages of this summary; for more detailed treatment we suggest the first two sections from Chapter 1 of Patrick Billingsley's "Convergence of Probability Measures").

Course content: Lecture notes will be posted before the start of the course (the first 3 lectures are to be complemented by problem sessions).

- Lecture 1:
  - First examples. Large deviation principle.
  - Cramér's theorem on $\mathbb{R}$: statement and two main ideas.
  - Legendre-Fenchel transform and its basic properties.
- Lecture 2:
  - Data compression. Entropy.
  - Method of types. Sanov's theorem.
  - Binary investigation.
- Lecture 3:
  - Varadhan's lemma and Bryc's inverse of it.
  - Applications: contraction principle and exponential tilting.
  - Curie-Weiss model I (deterministic external field).
- Lecture 4:
  - Gärtner-Ellis Theorem.
  - Curie-Weiss model II (random external fields).

What's next? This mini-course consists of only 4 lectures and is bound to have a very limited scope. Many important results, techniques, and exciting examples are still awaiting to be explored. The literature on large deviations is very large. There are many good books at various levels (predominantly at a much higher level than our course) and many sets of lecture notes on the web (at all levels). Some of the sources are referenced in lecture notes. Here we shall simply suggest the next step and give a (non-comprehensive) list of textbooks for in depth studies.

- Step 1: Frank den Hollander, *Large Deviations*, Fields Institute Monographs in mathematical Sciences, AMS, 2000, 143 pages.
  This is a relatively short and friendly introductory text which would be a natural expansion of this mini-course.
- Extended Step 1: Firas Rassoul-Agha, Timo Seppäläinen, *A Course on Large Deviations With an Introduction to Gibbs Measures* (Graduate Studies in Mathematics), AMS, 2015, 318 pp.
  This textbook is intended for students from different fields who are interested in statistical mechanics. The authors provide the necessary background in analysis, probability, and statistical mechanics within the main text and in a set of Appendices.
- An alternative path: Srinivasa R. S. Varadhan. *Large deviations*, Courant Lecture Notes in Mathematics, 27. AMS, 2016, 104 pp.
  This book assumes a good background in stochastic processes and focuses on three sets of examples.
  A natural path for a beginning graduate student could be to start with the first two courses by S.R.S. Varadhan in the same series: *Probability Theory* (Courant lecture notes 7), *Stochastic Processes* (Courant lecture notes 16), and then turn to large deviations.
- Amir Dembo and Ofer Zeitouni, *Large Deviations Techniques and Applications*, Springer, 2nd Edition, 1998, 396 pp.
  This is both a textbook with many examples and exercises as well as a convenient reference book intended for students and researches with different backgrounds.

The next list provides a sample of well-known classical monographs on large deviations and closely related topics.

- Jean-Dominique Deuschel, Daniel W. Stroock, *Large deviations.* Pure and Applied Mathematics, 137. Academic Press, 1989, 307 pp.
- Richard Ellis, *Entropy, large deviations, and statistical mechanics.* Grundlehren der Mathematischen Wissenschaften, 271. Springer, 1985, 364 pp.
- Mark I. Freidlin, Alexandre D. Wentzell. *Random perturbations of dynamical systems.* Grundlehren der Mathematischen Wissenschaften, 260. Springer, 1984, 326 pp.
- Srinivasa R. S. Varadhan, *Large deviations and applications.* CBMS-NSF Regional Conference Series in Applied Mathematics, 46. SIAM, 1984, 75 pp.