Contact geometry: From dimension three to higher dimensions

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Abstract

We discuss relations between the global geometry of closed contact manifolds and the geometry of compact symplectic Stein manifolds that they bound. The origin of these relations is the existence of open book decompositions adapted to contact structures.

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Three-dimensional contact geometry has experienced significant advancement during the last decade thanks to the development of adequate topological methods. Following the work of Bennequin [Be] and Eliashberg [El1], the theory of convex surfaces [Gi1] and the study of bypasses [Ho] have led to a complete classification of contact structures on some simple manifolds and, more recently, to a coarse classification on all closed manifolds [Co, HKM, CGH]. In fact, as we will show later, contact structures in dimension three are purely topological objects, like symplectic structures in dimension two. In precise terms, on any closed three-dimensional manifold V, the isotopy classes of the contact structures are in one-to-one correspondence with the isotopy and stabilization classes of open books in V, where the elementary stabilization operation is a positive plumbing [Gi2].

In higher dimensions, radically different methods have made it possible to show a similar correspondence [GM] and, beyond that, to reveal close links between the global

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geometry of closed contact manifolds and that of certain compact symplectic manifolds. The open books that are associated with a contact structure are indeed particular: their pages are compact Stein manifolds, their monodromy is a symplectic diffeomorphism with support in the interior and the elementary stabilization operation that unifies them is a positive Lagrangian plumbing. In addition, the essential tool for their construction is the positive bundle theory that Donaldson introduced and developed in symplectic geometry in [Do1, Do2] and Ibort, Martínez, and Presas adapted to contact geometry in [IMP].

A. Contact structures and open books

Throughout this text, V denotes a closed and oriented manifold. Hyperplane fields on V are cooriented, therefore also oriented since V is oriented. Such a hyperplane field ξ is the kernel of a 1-form α, called *an equation of* ξ, unique up to multiplication by a positive function. We say that ξ is a *contact structure* if $d\alpha$ at every point induces a symplectic form on ξ, i.e. if V is of dimension $2n+1$ and $(\alpha \wedge d\alpha)^n$ is at any point a volume element for the orientation of V.

On the other hand, an *open book* in V is a pair (K, θ) formed by the following objects:

- a codimension two submanifold $K \subset V$ with trivial normal bundle;
- a fibration $\theta \colon V \setminus K \to \mathbf{S}^1$ which, in a neighborhood $K \times \mathbf{D}^2$ of $K = K \times \{0\}$, coincides with the normal angular coordinate.

One can also view open books in a different way. Let $\phi \colon F \to F$ be a diffeomorphism of a compact manifold which restricts to identity near the boundary $K = \partial F$. Its suspension, namely the compact manifold

$$\Sigma(F, \phi) = \big(F \times [0,1]\big)/\!\sim, \quad \text{where } \ (p,1) \sim (\phi(p), 0),$$

is bounded by $K \times \mathbf{S}^1$ – because $\phi\,|_K = \mathrm{id}$ – and the closed manifold

$$\overline{\Sigma}(F, \phi) = \Sigma(F, \phi) \cup_\partial (K \times \mathbf{D}^2),$$

has an obvious open book. Moreover, any open book (K, θ) in V identifies V with $\overline{\Sigma}(F, \phi)$, where F is a fiber of θ (slightly shrunk) and ϕ is the first return map on F of a flow transverse to the fibers of θ and, near K, consisting of rotations around K. The diffeomorphism ϕ, defined only up to conjugation and isotopy, is the *monodromy* of (K, θ).

The upcoming discussion revolves around the following definition:

Definition 1 [Gi2, GM]**.** *A contact structure ξ on V is said to be* supported[1] *by an open book (K, θ) if it admits an equation α with the following properties:*

[1] Translator's note: The author, E. Giroux, uses the term *portée* which roughly translates to English as *carried*. However, for convenience, we will be translating the term as *supported* instead throughout the text.

- *α induces a contact form on K;*
- *$d\alpha$ induces a symplectic form on each fiber F of θ;*
- *The orientation on K defined by the contact form α coincides with its orientation as the boundary of the symplectic manifold $(F, d\alpha)$.*

Such a form α will be said to be adapted *to (K, θ).*

Example [GM]**.** Let $f\colon (\mathbf{C}^n, 0) \to (\mathbf{C}, 0)$ be a holomorphic function with an isolated critical point at the origin and H be the (singular) hypersurface $f^{-1}(0)$. There exists a smooth closed ball B around the origin in $\mathbf{C}^n$ and a foliation of $B \setminus \{0\}$ by strictly pseudoconvex spheres S_r, where $r \in]0, 1]$ and $S_1 = \partial B$, such that, for sufficiently small r, the following properties are satisfied:

- The sphere S_r is transverse to H, so that $K = H \cap S_r$ is a codimension two closed submanifold of S_r with trivial normal bundle;
- The map $\theta = \arg f\colon S_r \setminus K \to \mathbf{S}^1$ is a fibration that makes (K, θ) an open book;
- The open book (K, θ) supports the contact structure on S_r defined by the field of complex tangents.

In other words, each open book of the sphere, given by the Milnor fibration theorem, supports, up to isotopy, the standard contact structure.

B. Contact structures and open books in dimension three

In dimension three, various works have long since revealed connections between contact structures and open books without, however, establishing any formal link between them. In [TW], W. Thurston and H. Winkelnkemper construct contact forms on any closed manifold V from an open book of V. With the terms of Definition 1, they actually show that any open book in V supports a contact structure. In [Be], on the other hand, to transform his result on closed braids into a theorem in contact geometry, D. Bennequin highlights the following property: Any curve transverse to the standard contact structure ξ_0 in $\mathbf{R}^3$ – with the equation $dz + r^2 d\theta = 0$ – is isotopic, among transverse curves, to a closed braid i.e. a curve transverse to the open book formed by the z-axis and the angular coordinate θ. This property actually comes from the fact that this open book supports ξ_0. Lastly, in [To], I. Torisu clearly identifies the relationship between open books and Morse theory configurations considered in [Gi1] to study convex contact structures in the sense of [EG].

The first observation which shows the intimate connections imposed by the definition and follows from the stability of the contact structures is as follows:

Proposition 2 [Gi2]**.** *On a three-dimensional closed manifold, any two contact structures supported by the same open book are isotopic.*

As for the question of which contact structures have a supporting open book, the answer is simple:

Theorem 3 [Gi2]**.** *On a three-dimensional closed manifold, any contact structure is supported by an open book.*

However, as the Milnor fibration example illustrates, the open book that supports a given contact structure is far from unique - even up to isotopy. To understand this phenomenon, some definitions are useful.

Let $F \subset V$ be a compact surface with boundary and $C \subset F$ be a proper simple arc. We say that a compact surface $F' \subset V$ is obtained from F via *positive* (resp. *negative*) *plumbing* of an annulus along C if $F' = F \cup A$ where $A \subset V$ is an annulus with the following properties:

- $A \cap F$ is a regular neighborhood of C in F;
- A is included in a closed ball B whose intersection with F is only $A \cap F$ and the linking number of the two components of ∂A in B is 1 (resp. -1)

A result of J. Stallings states that, if (K, θ) is an open book in V and if F is the closure of a fiber of θ, then for any surface F' obtained from F by plumbing an annulus, there is an open book (K', θ') such that K' is the boundary of F' and F' is the closure of a fiber of θ'. In what follows, we will say that the open book (K', θ') and the knot K' themselves are obtained by plumbing from (K, θ) and K, respectively. In addition, we will say that an open book (K', θ') is a stabilization of another open book (K, θ) if it is obtained from (K, θ) by a finite sequence of positive plumbings .

Theorem 4 [Gi2]**.** *On a three-dimensional closed manifold, any two open books which support the same contact structure have isotopic stabilizations.*

Theorems 3 and 4 allow us to translate a number of questions on contact structures into questions on open books, in other words on diffeomorphisms of compact surfaces with boundary. In this sense, they are the analogues of the theorems of S. Donaldson [Do2] on Lefschetz pencils in four-dimensional symplectic manifolds. However, unlike those (in [Do2]), they admit purely topological proofs, the ideas of which are briefly described below, after having introduced the essential tool. It will be assumed that the reader is familiar with certain notions of three-dimensional contact geometry (overtwisted/tight contact structures, Thurston-Bennequin invariant of Legendrian curves, ξ-convex surfaces).

We say that a *polyhedral cell* in V is the image of a topological embedding of a compact convex Euclidean polyhedron. Such a cell has an affine structure induced by its parameterization and its *interior* is, by definition, the image of the intrinsic interior of the polyhedron, namely of its topological interior in its affine hull.

A *polyedral cellulation* of V here refers to a finite cover of V by polyhedral cells with the following properties:

- The interiors of cells form a partition of V;
- The boundary of each cell D is a union of cells D_j and the inclusions $D_j \to D$ are affine;
- The cells of dimension two (and less) are smooth, *i.e.* are images of smooth embeddings.

Polyhedral cellulations have the advantage over triangulations of being very easy to subdivide: any subdivision of a subcomplex is trivially extended. In addition, they play a key role in the proof of the Reidemeister-Singer theorem given in [Si] which serves as a guide to establish Theorem 4.

Sketch of the proof of Theorem 3. Let ξ be a contact structure on V. We first build a *contact cellulation* of (V, ξ), namely a polyhedral cellulation Δ with the following properties:

1) Each cell of dimension 1 is a Legendrian arc.
2) Each cell of dimension 2 is ξ-convex and the Thurston-Bennequin invariant of its boundary is -1.
3) Each cell of dimension 3 is contained in the domain of a Darboux chart.

We then thicken the 1-skeleton L of Δ into a compact surface $\hat{F}$ (almost) tangent to ξ along L and we choose a regular neighborhood W of L small enough so that $F = \hat{F} \cap W$ is a surface properly embedded in W. Taking a smaller W, ξ admits an equation α satisfying the following conditions:

- $d\alpha$ induces an area form on F;
- α is nonsingular on $K = \partial F$ and orients K as the boundary of $(F, d\alpha)$.

On the other hand, for any cell D of dimension 2, the property 2) says that the boundary of $D \cap (V \setminus \operatorname{Int} W)$ intersects K at two points (up to isotopy). As a result, there exists a fibration $\theta \colon V \setminus K \to \mathbf{S}^1$ whose fiber is $\operatorname{Int} F$. Trimming off W, we can assume that W is a union of fibers of θ on which $d\alpha$ induces an area form. It remains to show that ξ is isotopic, relative to W, to a contact structure supported by (K, θ). The key point is that ξ is tight on $W^* = V \setminus \operatorname{Int} W$ and that ∂W^* is a ξ-convex surface whose dividing set is provided by K. □

Steps of the proof of Theorem 4. Let Δ be a contact cellulation of (V, ξ). We will say here that an open book supporting (K, θ) is *associated* with Δ if, as in the proof of Theorem 3, one of the fibers of θ contains the 1-skeleton of Δ and retracts on it by a contact isotopy. By imitating [Si], we first show that any supporting open book admits a stabilization associated with a contact cellulation. We thus return to consider the case of two supporting open books associated with contact cellulations Δ_0 and Δ_1 in general position. According to [Si], Δ_0 and Δ_1 have a common subdivision Δ_2 which is obtained, from both Δ_0 and Δ_1, by bisections. We then deform Δ_2, relative to the union of the 1-skeletons of Δ_0 and Δ_1, into a cellulation satisfying the properties 1) and 3) of the contact cellulations and having ξ-convex 2-cells. Then it suffices to subdivide the 2-skeleton of Δ_2 to obtain a contact cellulation Δ and finally we show that the open book associated with Δ is a stabilization of those associated with Δ_0 and Δ_1. □

We now discuss some corollaries of Theorem 3 and Theorem 4. First we recall that a theorem by M. Hilden and J. Montesinos states that any closed three-dimensional manifold V is a three-fold cover of the sphere $\mathbf{S}^3$ simply branched over a link (simply means that the local degree at the branched points in V is two). The same result is holds for closed contact manifolds:

Corollary 5 [Gi2]**.** *Any three-dimensional closed contact manifold is a three-fold cover of the standard contact sphere $(\mathbf{S}^3, \xi_0)$ simply branched over a link transverse to ξ_0.*

Another corollary concerns the dynamics of Reeb flows. A *Reeb flow* on a contact manifold is a flow that preserves the contact structure while being transverse to it and pointing to the positive side of it. A typical example is the geodesic flow on the unit cotangent bundle of a Riemannian manifold. The Reeb flows of a given contact structure ξ are in bijection with the equations of ξ: To any form α, there corresponds a unique vector field ∇_α which generates the kernel of $d\alpha$ and on which α is 1. Taking an equation of ξ adapted to a supporting open book, we get:

Corollary 6 [Gi2]**.** *On any three-dimensional closed contact manifold, there exists a Reeb flow that admits a Poincaré-Birkhoff section, i.e. a compact surface, which meets all the orbits, whose interior is transverse to the flow and boundary components are periodic orbits.*

In fact, it is not excluded that any Reeb flow admits such a section [HWZ] (This would imply the Weinstein's conjecture which states that every Reeb flow has a periodic orbit.) but this is a problem of a different nature, certainly inaccessible by topological methods.

In view of Theorems 3 and 4, a natural question is how to read off the monodromy of its supporting open book if a contact structure is tight or fillable in any sense. The only known answer concerns *holomorphically fillable* contact structures, i.e. realizable as fields of complex tangents on the boundary of compact Stein manifolds. The following corollary specifies a result of A. Loi and R. Piergallini:

Corollary 7 [LP, Gi2]**.** *A contact structure on a three-dimensional closed manifold is holomorphically fillable if and only if it is supported by an open book whose monodromy is a product of right-handed Dehn twists.*

Finally, we give a corollary of pure knot theory. Here we call *fibered link* in V any oriented link K for which there exists a fibration $\theta\colon V \setminus K \to \mathbf{S}^1$ which makes (K, θ) an open book and induces the prescribed orientation on K. When V is a homology sphere, a theorem of F. Waldhausen assures that this fibration, if exists, is unique up to isotopy. The following result answers a question posed by J. Harer in [Ha]:

Corollary 8 [Gi2]**.** *Any two fibered links in a homology sphere V are obtained from each other by a series of plumbings and unplumbings (inverse operation).*

Proof. A trivialization of V being chosen, the homotopy classes of plane fields tangent to V are identified by their *Hopf invariant*, i.e. the linking number of the fibers of the corresponding map $V \to \mathbf{S}^2$. We then consider any open book (K, θ) in V and denote by (K', θ') an open book obtained from (K, θ) by a negative plumbing. The following three observations prove the Corollary:

- any contact structure ξ' supported by (K', θ') is overtwisted because the core of the plumbed annulus is isotopic to a Legendrian unknot in (V, ξ') whose Thurston-Bennequin invariant is $+1$;
- the Hopf invariant of ξ' is one unit greater than that of the contact structures supported by (K, θ) (see [NR]);
- if two overtwisted contact structures have the same Hopf invariant, then they are isotopic according to [El1] and any two open books supporting them therefore have isotopic stabilizations.

This argument, moreover, limits the number of negative (un)plumbings needed to move from one open book to another by $h + 2$, where h denotes the difference between the corresponding Hopf invariants. □

C. Contact structures and open books in higher dimensions

In dimension greater than three, open books supporting contact structures are not arbitrary: Their fibers have a symplectic structure invariant by the monodromy. To clarify this point, some definitions are useful.

Let F be a compact manifolds with boundary $K = \partial F$. An exact symplectic form ω on $\operatorname{Int} F$ is *convex at infinity* if there exists on $\operatorname{Int} F$ a Liouville vector field (vector field ω-dual to primitive of ω) which is transverse to all hypersurfaces $K \times \{t\}$, $t \in (0, 1]$, where

$K \times [0,1]$ is a collar neighborhood of $K = K \times \{0\}$. It is further said that $(\operatorname{Int} F, \omega)$ is a *Weinstein manifold* [EG] if there is such a Liouville vector field which, moreover, is the (pseudo) gradient of a Morse function $F \to \mathbf{R}$ constant and without critical points on K. The typical example of a Weinstein manifold is the interior of a *compact Stein manifold*. We thereby name any compact complex manifold F which admits a strictly pluri-subharmonic function $f\colon F \to \mathbf{R}$ constant and without critical points on the boundary. The 2-form $i\partial\overline{\partial} f$ then defines a symplectic structure. In fact, it follows from the work of Y. Eliashberg [El2] that any Weinstein manifold is symplectically diffeomorphic to such a compact Stein manifold.

Now if α is a contact form adapted to an open book (K, θ), its differential $d\alpha$ induces an exact symplectic structure convex at infinity on each fiber of θ. This structure depends on the choice of α but its completion [EG] is well-defined up to isotopy. The theorem of W. Thurston and H. Winkelnkemper and Proposition 2 then extend to higher dimensions:

Proposition 9 [GM]**.** *Let F be a compact manifold with, on $\operatorname{Int} F$, an exact symplectic form convex at infinity and let $\phi\colon F \to F$ be a symplectic diffeomorphism equal to the identity near $K = \partial F$. Then, there exists a contact structure on $\overline{\Sigma}(F, \phi)$ supported by the obvious open book. In addition, any two contact structures, supported by the same open book, which induce symplectic structures on its pages having isotopic completions are isotopic.*

As for Theorem 3, it generalizes as follows:

Theorem 10 [GM]**.** *Any contact structure on a closed manifold V is supported by an open book, each fiber of which is a Weinstein manifold.*

Sketch of the proof. Let ξ be a contact structure, α an equation of ξ, and J an almost complex structure on ξ compatible with $d\alpha\,|_\xi$. We denote by ∇_α the Reeb vector field associated to α and g a Riemannian metric on V which equals $d\alpha(\cdot, J\cdot)$ on ξ and makes ∇_α unitary and orthogonal to ξ. In elementary terms, the main theorem of [IMP] shows that there exist constants $C, \eta > 0$ and functions $s_k\colon V \to \mathbf{C}$, $k \geq 1$, satisfying the following conditions:

- At any point of V,

$$|s_k(p)| \leq C, \quad |ds_k - iks_k\alpha| \leq Ck^{1/2} \quad \text{and} \quad |\overline{\partial}_\xi s_k| \leq C\,;$$

- At any point p where $|s_k(p)| \leq \eta$,

$$|\partial_\xi s_k(p)| \geq \eta k^{1/2}\,.$$

(Here, $\partial_\xi s_k$ and $\overline{\partial}_\xi s_k$ are J-linear and J-antilinear, respectively, parts of $ds_k\,|_\xi$.) In more meaningful terms, the functions s_k are approximately holomorphic and equitransversal sections of the bundle $L^{\otimes k} \to V$, where L is the trivial Hermitian bundle $V \times \mathbf{C} \to V$ provided with the unitary connection defined by the form $-i\alpha$. The estimations above, first of all, imply that for $|w| \leq \eta$, the set $K_w = s_k^{-1}(w)$ is a submanifold and the form α_w induced from α on K_w is a contact form (see [IMP]). In fact, α_w is nonsingular for sufficiently large k since its kernel is equal to the kernel of $ds_k\,|_\xi$ and that $|\partial_\xi s_k| \geq \eta k^{1/2}$ while $|\overline{\partial}_\xi s_k| \leq C$. Better, these inequities show that, for k large, the kernel of α_w is close to a J-complex subspace of ξ so that $d\alpha_w$ is nondegenerate on it.

The following observation is that the map $\arg s_k \colon V \setminus K \to \mathbf{S}^1$ is a fibration whose fibers are transverse to the Reeb vector field ∇_α at any point where $|s_k| \geq \eta$. To see this, we note that the estimation on $ds_k - iks_k\alpha$ implies that

$$|ds_k(\nabla_\alpha) - iks_k| \leq Ck^{1/2}.$$

Therefore, at any point p where $|s_k(p)| \geq \eta$ and for k sufficiently large, $ds_k(p)\,(\nabla_\alpha)$ is close to $iks_k(p)$, *i.e.* is non-zero and almost orthogonal to $s_k(p)$. As a result, the submanifolds

$$s_k^{-1}(R_\theta), \quad \text{where} \quad R_\theta = \{re^{i\theta},\ r > \eta\},$$

are transverse to the Reeb vector field ∇_α.

These arguments show that the open book $(K = K_0,\ \theta = \arg s_k)$, for k large enough, supports the contact structure $\xi = \ker \alpha$. It remains to verify that the fibers of θ are Weinstein manifolds. For simplicity, we prove below the analogous assertion in symplectic geometry. □

Proposition 11. *Let W be a closed manifold, ω a symplectic form on W, and H_k a symplectic submanifold of W Poincaré dual to $k[\omega]$ obtained by the Donaldson construction* [Do1]*, from a Hermitian line bundle L equipped with a connection having curvature $-i\omega$. For sufficiently large k, $(W \setminus H_k, \omega)$ is a Weinstein manifold.*

Proof. Using the arguments of [Do2], we can suppose that H_k is the zero set of a section $s_k \colon V \to L^{\otimes k}$ which satisfies, at any point of W,

$$|\overline{\partial}_k s_k| \leq c|\partial_k s_k| \quad \text{with} \quad c < \frac{1}{\sqrt{2}}.$$

In the trivialization of $L^{\otimes k}$ given over $W \setminus H_k$ by the unitary section $u = s_k/|s_k|$, the connection is defined by a 1-form $-i\lambda$ where $d\lambda = k\omega$. If we set $s_k = \varphi u$, the inequality above gives

$$|d\varphi/\varphi + J^*\lambda| < |d\varphi/\varphi - J^*\lambda|,$$

which shows that $J^*\lambda$ is further from $d\varphi/\varphi$ than from $-d\varphi/\varphi$. The Liouville vector field dual to λ is then a pseudogradient vector field of $\log \varphi$. □

As in dimension three, the open book supporting a given contact structure is not unique. We describe in [GM] a *plumbing operation along a Lagrangian disk* – in which Dehn-Seidel twists replace Dehn twists – which makes it possible to establish analogues of Theorem 4 and Corollary 7. These results bring the study of contact structures back to those of symplectic diffeomorphisms of compact Stein manifolds which are the identity near the boundary. They may thus allow us to bring together the work of Y. Eliashberg, H. Hofer and A. Givental on symplectic field theory with those of, for example, P. Seidel on Floer homology and symplectic diffeomorphism groups. One may also ask if Theorem 10 hides obstructions to the existence of a contact structure on closed manifolds. According to [Qu], any closed manifold V of dimension $2n+1$ has an open book each fiber of which has the homotopy type of a cellular complex of dimension n. It is probable that, if V admits a tangent hyperplane field equipped with an almost complex structure, there exists such an open book for which each fiber is an almost complex manifold and is therefore, according to [El2], the interior of a compact Stein manifold. All the difficulty would therefore really be to realize the monodromy by a symplectic diffeomorphism... In this order of thought, here is a concrete corollary of Theorem 10 obtained by F. Bourgeois which shows, in response to an old question, that any odd-dimensional torus has a contact structure:

Corollary 12 [Bo]**.** *If a closed manifold V admits a contact structure, then so does $V \times T^2$.*

Proof. Let ξ be a contact structure on V and α be an equation of ξ supported by an open book (K, θ) and let $N = K \times \mathbf{D}^2$ be a neighborhood of $K = K \times \{0\}$ where θ is the normal angular coordinate. We denote the normal radial coordinate in N by r and set

$$\tilde{\alpha} = \alpha + f(r)(\cos\theta \, dx_1 - sin\theta \, dx_2), \qquad (x_1, x_2) \in \mathbf{T}^2 = \mathbf{R}^2/\mathbf{Z}^2,$$

where $f(r) = r$ for $r \leq r_0$ and $f(r) = 1$ for $r \geq 2r_0$ and satisfies $f'(r) \geq 0$. A calculation shows that, if we choose r_0 small enough, $\tilde{\alpha}$ is a contact form on $V \times \mathbf{T}^2$. □

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