Errata to "Stable ergodicity of skew products"

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The description of the center bunching property on pages 861–862 is wrong. Center bunching requires that both $\|T_pF|_{E^c_F}\|$ and $m(T_pF\|_{E^c_F})$ should be close enough to 1. Having the ratio $\mu_c = 1$ does not imply center bunching. But center bunching certainly holds if the action of Tf on E^c is isometric, as is the case for the skew product examples considered in the paper.

In lines 1 and 4 of 871 (bottom of page 14 in the tex version), (x_0, g_0) should be changed to (x_0, e).

The following description of the partition $\mathcal{P}$ in Section 3 may be clearer than the one given in the paper: $\mathcal{P}$ consists of the level sets of the function $\eta : M \times G \to H\backslash G$ defined by $\eta(x,g) = \Phi(x)^{-1}g$.

(In the paper $\mathcal{P}$ is described as consisting of the set $P = \bigcup_{x\in M}\{x\} \times \Phi(x)$ and its right translates. See the bottom of page 863 or page 6 of the tex version.)

The partition into level sets of η is right invariant. This follows because right multiplication by a constant respects the partition of G into cosets belonging to $H\backslash G$, in the sense that each coset from $H\backslash G$ is carried to another coset from $H\backslash G$. We have

$$\begin{aligned}\eta(x_1,g_1) = \eta(x_2,g_2) &\Leftrightarrow \Phi^{-1}(x_1)g_1 = \Phi^{-1}(x_2)g_2 \\ &\Leftrightarrow \Phi^{-1}(x_1)g_1g = \Phi^{-1}(x_2)g_2g \\ &\Leftrightarrow \eta(x_1,g_1g) = \eta(x_2,g_2g),\end{aligned}$$

for any g.

We now show that $P = \eta^{-1}(H)$, where P is the set defined above. In order to do this we show that $\Phi(x) = \{g : \eta(x,g) = H\}$ for each x. Suppose that $\Phi(x) = g(x)H$. Then $\Phi^{-1}(x) = Hg(x)^{-1}$ and

$$\begin{aligned}\eta(x,g) = \eta(x,g') &\Leftrightarrow \Phi^{-1}(x)g = \Phi^{-1}(x)g' \\ &\Leftrightarrow Hg(x)^{-1}g = Hg(x)^{-1}g' \\ &\Leftrightarrow g(x)^{-1}g'g^{-1}g(x) \in H \\ &\Leftrightarrow g'g^{-1} \in g(x)Hg(x)^{-1} \\ &\Leftrightarrow g' \in g(x)Hg(x)^{-1}g.\end{aligned}$$

It is clear that the intersection with $\{x\} \times G$ of the level set of η containing (x, g) is $g(x)Hg(x)^{-1}g$. In particular $\Phi(x) = g(x)Hg(x)^{-1}g(x)$ is a level set in $\{x\} \times G$. This level set contains $g(x)$ and maps to H.

Thus $P = \bigcup_{x \in M} \{x\} \times \Phi(x)$ is the inverse image of H. This shows that the descriptions of $\mathcal{P}$ given here and in the paper are equivalent.