NORTHWESTERN MASTERCLASS
HEEGAARD FLOER LECTURE SERIES
HOMEWORK 2

(1) Prove that $|H_1(Y)| = |T_\alpha \cdot T_\beta|$ (if the left hand side is finite), where $\cdot$ denotes the algebraic intersection number.

(2) Let K be a knot in S^3. Show that the following procedure produces a Heegaard diagram for (some) surgery on K:
  (a) Start with a doubly-pointed Heegaard diagram $(\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta}, z, w)$ for $K \subset S^3$.
  (b) Add a handle to Σ connecting z and w.
  (c) Add a new α-circle running through the handle once.
  (d) Add a new β-circle running through the handle once.

  (The framing of the surgery depends on how you add the α- and β-curves. How can you produce a framing of your choice?)

(3) Show that $\mathrm{Sym}^n(\mathbb{C}) \cong \mathbb{C}^n$. (Hint: given n unordered points in $\mathbb{C}$, there is a unique monic, degree n polynomial with these points as roots. Consider the coefficients of this polynomial.)

  Show that for Σ an orientable surface, $\mathrm{Sym}^n(\Sigma)$ is a smooth manifold.

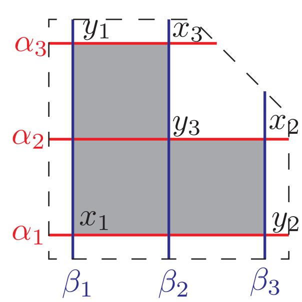
(4) We asserted: let S be a surface with boundary and $u_\Sigma\colon S \to \Sigma$, $u_D\colon S \to \mathbb{D}^2$ holomorphic maps. Suppose that u_D is a g-fold branched cover. Then the map

$$\mathbb{D}^2 \to \mathrm{Sym}^g(\Sigma)$$
$$p \mapsto u_\Sigma(u_D^{-1}(p))$$

  is $\mathrm{Sym}^g(j)$-holomorphic. Prove this. (As a consequence, this map is continuous.)

(5) As we discussed in the lecture, if you see a rectangle in a Heegaard diagram (with all four edges lying on different circles) then there is a corresponding holomorphic curve in $\mathrm{Sym}^g(\Sigma)$. The analogous fact holds for $2n$-gons, for any n. Prove it.

(6) Consider the following element of $\pi_2(\{x_1, x_2, x_3\}, \{y_1, y_2, y_3\})$:



  (This is a piece of a Heegaard diagram; for instance, you can find pieces like this inside grid diagrams.) There is a corresponding 1-parameter family of holomorphic disks. Describe it. (Hint: the family comes from making a slit in the domain along either an α- or β-curve.) This corresponds to a cancellation in $\partial^2\{x_1, x_2, x_3\}$; which one?

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