MARGINAL PRODUCT OF LABOR AND CAPITAL

Assume $Q = f(L, K)$ is the production function where the amount produced is given as a function of the labor and capital used. For example, for the Cobb-Douglas production function $Q = f(L, K) = A\,L^a\,K^b$. For a given amount of labor and capital, the ratio $\dfrac{Q}{K}$ is the average amount of production for one unit of capital. On the other hand the change in the production when the labor is fixed and the capital is changed from K to $K + \Delta K$ is $\Delta Q = f(L, K + \Delta K) - f(L, K)$. Dividing this quantity by ΔK gives the change in the production per unit change in capital,

$$\frac{\Delta Q}{\Delta K} = \frac{f(L, K + \Delta K) - f(L, K)}{\Delta K}.$$

Usually, we take the limit with infinitesimal changes in capital, or taking the limit as ΔL goes to zero we get

$$\frac{\partial Q}{\partial K},$$

which is called the *marginal product of capital*. The term "marginal" is used because it measures the change of production with a small change of capital.

In the same way,

$$\frac{\partial Q}{\partial L},$$

which is called the *marginal product of labor*.

For the Cobb-Douglas production function

$$\frac{\partial Q}{\partial K} = b\,A\,L^a\,K^{b-1} = \frac{b\,Q}{K} \qquad \text{and}$$
$$\frac{\partial Q}{\partial L} = a\,A\,L^{a-1}\,K^b = \frac{a\,Q}{K}.$$

Thus, for the Cobb-Douglas production function, the marginal product of capital (resp. labor) is a constant times the average product of capital (resp. labor).

These marginal rates depend on the units used for measuring the quantities. The ratio of the amount of change to the amount held does not depend on the units, ${}^{\Delta Q}\!/_{Q}$, and can be interpreted as the "percentage change" of the quantity. If we use the ratios of the percentage changes, then we get a quantity which is independent of the units. For example,

$$\frac{{}^{\Delta Q}\!/_{Q}}{{}^{\Delta K}\!/_{K}} = \frac{K}{Q} \cdot \frac{\Delta Q}{\Delta K}$$

does not depend on the units. Taking the limit with small changes in the change of capital, we get

$$\epsilon = \frac{K}{Q} \cdot \frac{\partial Q}{\partial K} = \frac{\dfrac{\partial Q}{\partial K}}{\dfrac{Q}{K}},$$

which is called the *capital elasticity of product*. Notice that it equals the ratio of the marginal product of capital to the average product of capital. If $\epsilon < 1$, then the next small change in capital makes less change in the output than the average capital per unit capital, i.e., there is diminishing returns on capital. For this situation, the production function is called *inelastic*. For the Cobb-Douglas production functions, the capital elasticity of product equals the exponent b. So, when $0 < b < 1$, the production function is inelastic.