CORRECTIONS TO MATH 313-1 NOTES

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page 2 (line 5) T^2 has two similar "tents" on the interval $[0, 1]$.

page 47 (line -1) $\hat{T}(\theta) = 2(\pi - \theta)$ if $\dfrac{\pi}{2} \le \theta \le \pi$.

page 67 (line 19) $f(\mathbf{I}_{s_{j-1}}) \supset \mathbf{I}_{s_j}$ for $1 \le j \le n$.

page 67 (line 24-5) An allowable periodic sequence $s_0, \dots, s_n$ is called *reducible* provided $n = mp$ with $p > 1$ and the sequence of symbols $s_0, \dots, s_{n-1}$ is just the sequence $s_0, \dots, s_{m-1}$ repeated p times.

page 68 (line 13) $\mathbf{J}_n = \mathbf{I}_{s_n}$,

page 74 (line 12) by $p\sqrt{2}/q$, where p and q are integers.

page 81-2 (There is a bad letter for the variable in the discussion of the Logistic map) The logistic function $G(y) = g_4(y)$, for $a = 4$, also takes the interval $[0, 1]$ to itself since $G(0.5) = 1$. The results for the logistic map $G(y)$ are the same as the tent map. Let $\mathbf{I}_L$ and $\mathbf{I}_R$ be the same are for the tent map. Given a symbol sequence $\mathbf{s}$, define the intervals

$$\mathbf{I}^G_{s_0 \dots s_n} = \{\, y : G^j(y) \in \mathbf{I}_{s_j} \text{ for } 0 \le j \le n \,\}.$$

We showed in Proposition 2.6.4 that the map

$$y = C(x) = \sin^2\left(\frac{\pi x}{2}\right) = (1 - \cos(\pi\, x))/2$$

is a conjugacy from $T(x)$ to $G(y)$. Because there is a conjugacy C between T and G with $C([0, 0.5]) = [0, 0.5]$ and $C([0.5, 1]) = [0.5, 1]$,

$$\begin{aligned}
\mathbf{I}^G_{s_0 \dots s_n} &= \bigcap_{j=0}^{n} G^{-j}(\mathbf{I}^G_{s_j}) \\
&= \bigcap_{j=0}^{n} C \circ T^{-j} \circ C^{-1}(\mathbf{I}^G_{s_j}) \\
&= C\left(\bigcap_{j=0}^{n} T^{-j}(\mathbf{I}^T_{s_j})\right) \\
&= C(\mathbf{I}^T_{s_0 \dots s_n}),
\end{aligned}$$

where $\mathbf{I}^T_{s_0 \dots s_n}$ is the interval for the tent map. There are bounds on the derivative of the conjugacy equation,

$$\begin{aligned}
C'(x) &= \frac{\pi}{2}\, \sin(\pi x) \\
|C'(x)| &\le \frac{\pi}{2}.
\end{aligned}$$

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Let x_0 and x_1 be the end points of the interval $\mathbf{I}^T_{s_0\dots s_n}$, so $|x_1 - x_0| = 2^{-n-1}$, and $y_0 = C(x_0)$ and $y_1 = C(x_1)$ be the corresponding end points of the interval $\mathbf{I}^G_{s_0\dots s_n}$. By the Mean Value Theorem, there is a point x_2 between x_0 and x_1, such that

$$\begin{aligned} y_1 - y_0 &= C(x_1) - C(x_0) \\ &= C'(x_2)\,(x_1 - x_0) \\ |y_1 - y_0| &= |C'(x_2)|\,|x_1 - x_0| \\ &\le \frac{\pi}{2}\,|x_1 - x_0| \\ &= \pi\, 2^{-n-2}. \end{aligned}$$

Since these intervals are going to zero in length as n goes to infinity,

$$\bigcap_{j=0}^{\infty} G^{-j}(\mathbf{I}_{s_j})$$

is a single point which we define as $k(\mathbf{s})$.

page 95 (line 2)

$$\Lambda_g = \bigcap_{n\ge 0} g^{-n}([0,1])$$

page 97 (line 1) The length of $\mathbf{I}_L$ is c and the length of $\mathbf{I}_R$ is $1-c$, so

page 97 (Theorem 3.5.4a) where L is the maximum of $p_1 - p_0, \dots p_k - p_{k-1}$.

page 97 (Theorem 3.5.4c) If $k(\mathbf{s})$ is neither periodic nor eventually periodic ...

page 106 (line 14) $R^3_{22.2,1}(0.05) \le 0.05$

page 106 (line 18) the interval from 0 up to 8.1

page109 (line 3) Since $f^{2^{q-1}}(x_q) = g(x_q) \ne x_q, \dots$

page 114 (3.3.1) Let $f(x) = 3\,x \pmod 1$ be the tripling map.

a. Prove that if two distinct points x_0 and x_0' are within a distance 1/6, then their iterates are at least three times as far apart.
b. Find a pair of point whose distance is not tripled by the map.
c. Show that f has sensitive dependence on initial conditions.

page 114 (3.3.2) Let p be a fixed point for f such that $|f'(p)| > 1$. Prove that f has sensitive dependence on initial conditions at p.

page 125 (Example 4.2.15) The number 0.08 should be 0.8. This mistake occurs several (at least 4) places in this example.

page 126 (line 1) Then, since $f^2([-0.3125, 0)) \supset (0, 0.2]$, $\mathbf{A}$ must contain the entire interval $(0, 0.2]$.

page 146 (line -9) We *show* that we can use this r ...

page 147 (line 6-10) Taking the second iterate,

$$\ell(\mathbf{J}_{k+2}) \ge \begin{cases} \lambda^2\,\ell(\mathbf{J}_k) & \text{if } c \notin f(\mathbf{J}_k) \cup f(\mathbf{J}_{k+1}) \\ \dfrac{\lambda^2}{2}\,\ell(\mathbf{J}_k) & \text{if } c \notin f(\mathbf{J}_k) \cap f(\mathbf{J}_{k+1}) \\ \dfrac{\lambda^2}{4}\,\ell(\mathbf{J}_k) & \text{if } c \in f(\mathbf{J}_k) \cap f(\mathbf{J}_{k+1}). \end{cases}$$

The first case assumes that c is in neither $f(\mathbf{J}_k)$ nor $f(\mathbf{J}_{k+1})$; the second case assumes that c is not in both $f(\mathbf{J}_k)$ and $f(\mathbf{J}_{k+1})$; the last case assumes that c is in both $f(\mathbf{J}_k)$ and $f(\mathbf{J}_{k+1})$, i.e., in two successive iterates. Since

$\lambda > \sqrt{2}$, $\lambda^2/2 > 1$, and the length of every second iterate grows until two successive iterates $f(\mathbf{J}_{n-4})$ and $f(\mathbf{J}_{n-3})$ contain c.

By assumption (ii) on the function, $f(c^-) = b$ and $f(c^+) = a$, $f(\mathbf{J}_{n-3})$ contains either the interval $(a, c]$ or $[c, b)$.