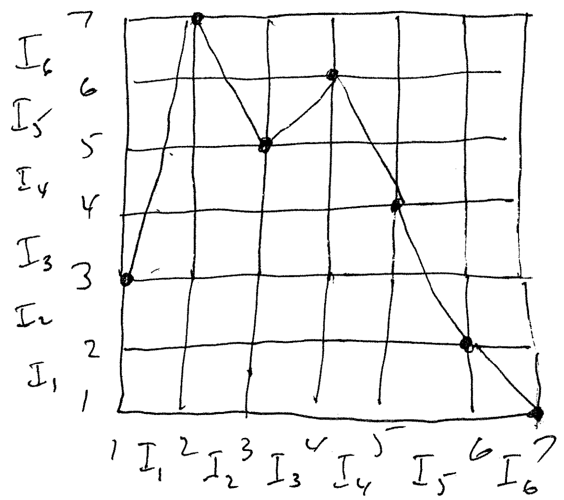
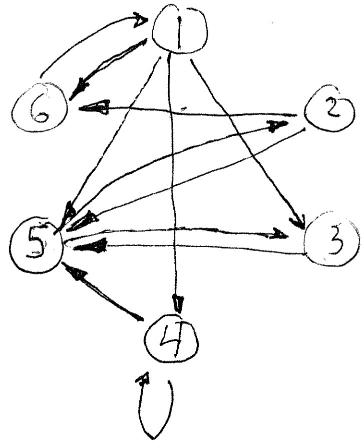
Math 313-1 Test 2 2/25/03

(1)





Transition Graph

$1 \to 3$
$1 \to 4$
$1 \to 5$
$2 \to 5$
$2 \to 6$
$3 \to 5$
$4 \to 4$
$4 \to 5$
$5 \to 2$
$5 \to 3$
$6 \to 1$

(b)

n	Symbol	Yes/No
1	4^∞	Yes
2	$(25)^\infty$, $(35)^\infty$, $(16)^\infty$	Yes
3	——	No
4	$(1526)^\infty$, $(3525)^\infty$	Yes
5	$(14526)^\infty$, $(13526)^\infty$	
6	$(144526)^\infty$, $(161526)^\infty$, $(152526)^\infty$	Yes
7	$(1353526)^\infty$, $(1444526)^\infty$, $(1352526)^\infty$	Yes

All other periods by Sharkovskii

(2) $Q(x) = 4x \pmod 1$.

Show that $|Q(x)-Q(y)| \geq 4|x-y|$ as long as $|x-y| \leq \frac{1}{8}$.

If they are in the same subinterval, $[0,\frac{1}{4})$, $[\frac{1}{4},\frac{1}{2})$, $[\frac{1}{2},\frac{3}{4})$, $[\frac{3}{4},1)$ the distance is multiplied by 4.

Assume $x<y$ and they are in adjacent intervals.

$Q(x) = 4x-k \qquad Q(y) = 4y-k-1$

since y in next interval.

$Q(x)-Q(y) = 1-4y+4x = 1-4(y-x)$.

So this $\overset{?}{\geq} 4|y-x|$.

$1 \overset{?}{\geq} 8|y-x|$

$\frac{1}{8} \geq |y-x|$.

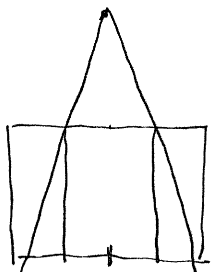
So true as long as $|y-x| \leq \frac{1}{8}$.

Thus distance are multiplied by 4 until the iterates are at least a distance $\frac{1}{8}$ apart.

This shows Q is expansive so has sensitive dependence on initial conditions with $r = \frac{1}{8}$.

Remark: The intervals $[0,\frac{1}{4})$, $[\frac{1}{4},\frac{1}{2})$, $[\frac{1}{2},\frac{3}{4})$ $[\frac{3}{4},1)$, $[1]$ are not closed ~~so~~ (map is not continuous) so the proof in the book using symbolic dynamics does not apply directly.

(3)



(b) K_3 has $2^3 = 8$ intervals each of length $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$

(c) K does not contain the interval $\left(\frac{1}{4}, \frac{3}{4}\right)$ which uses 1 & 2 in the expansion. Similar at the smaller levels. Therefore all members in K can be expressed using 0s & 3s.

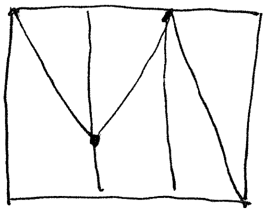
(d) $\frac{3}{4} + \frac{3}{4^3} + \frac{3}{4^5} + \cdots = \frac{3}{4}\left[1 + \frac{1}{16} + \frac{1}{16^2} + \cdots\right] = \frac{3}{4} \cdot \frac{1}{\frac{15}{16}} = \frac{4}{5}$

is in K but not an end point.

Also $\frac{3}{4^2} + \frac{3}{4^4} + \cdots = \frac{3}{16}\left[1 + \frac{1}{16} + \frac{1}{16^2} + \cdots\right] = \frac{3}{16} \cdot \frac{1}{\frac{15}{16}} = \frac{1}{5}$.

There are many other points.

(4) (a)

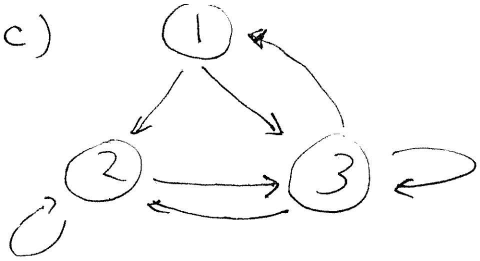


(b) Markov partition

$I_1 = [0, \frac{1}{3}]$, $I_2 = [\frac{1}{3}, \frac{2}{3}]$ $I_3 = [\frac{2}{3}, 1]$.

Expanding factor of 2

(c)



(d) The transition graph is irreducible (can get from any vertex to any other vertex.)

$f([0,1]) = [0, 1]$.

~~There are~~

This map is piecewise expanding and continuous.

Therefore this map is topologically transitive.