

1. (25 Points) Find a (pure strategy) Nash equilibrium for the bimatrix game where the payoff for player one is given by the matrix A and the payoff for player two is given by the matrix B :

$$A = \begin{pmatrix} 3 & 4 & 1 \\ 5 & 3 & 2 \\ 1 & 1 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 4 \end{pmatrix}.$$

2. (25 Points) Consider the bimatrix game with

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}.$$

Find the (interior) Nash equilibrium with mixed strategies.

3. (25 Points) For quick game of dueling, suppose the players P_1 and P_2 start only three steps apart, so there are only four stages to the game. First, P_1 has to decide whether to shoot or wait. If P_1 shoots, the payoffs are $(1, 0)$ if P_1 hits and $(0, 1)$ if P_1 misses. If the decision is to wait, then P_2 takes one step forward and faces the same decision, now at two steps apart. The payoffs for hitting or missing are the same with the players interchanged. If P_2 waits, then P_1 takes one step forward and faces the same decision, now at one step apart. Again, if P_1 waits, then P_2 takes one step forward and faces the same decision, now at zero steps apart. Suppose the probability of each player hitting the other player at the different distances is given by the following chart: here d is the number of steps apart, $p_1(d)$ is the probability of P_1 hitting at distance d , and $p_2(d)$ is the probability of P_2 hitting at distance d .

d	p_1	p_2
3	0.1	0.2
2	0.3	0.4
1	0.7	0.8
0	1.0	1.0

- (a) Set up the game tree.
 (b) Use the backward induction method to find the subgame perfect Nash equilibrium. Indicate the expected payoff at each node and the strategy profile each player follows.
4. Consider the bargaining problem with the feasible allocation set

$$s_1 + s_2 \leq 1, \quad s_1 \geq 0, \quad s_2 \geq 0.$$

Suppose the utility functions are $u_1(s_1, s_2) = \sqrt{s_1}$ and $u_2(s_1, s_2) = s_2$.

- (a) (25 Points) What is the Nash solution of the bargaining? What ~~is~~ ^{values} the ~~split~~ ^{are} of the s_j 's and the u_j 's at the solution?
 (b) (25 Points) What is the Kalai-Smorodinsky solution of the bargaining? What ~~is~~ ^{are} the ~~split~~ ^{values} of the s_j 's and the u_j 's at the solution?

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