

① Case (i) $E = u(10, 700) = 10,700^2 = 114,490,000$

Case (ii)

$$\begin{aligned} E &= \frac{1}{3} u(9,000) + \frac{1}{3} u(11,000) + \frac{1}{3} u(12,000) \\ &= \frac{1}{3} (81,000,000 + 121,000,000 + 144,000,000) \\ &= 346,000,000/3 = 115,333,333. \end{aligned}$$

Choice (ii) is better choice

② $A = \begin{pmatrix} 1 & 5 & 1 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{pmatrix}$ Choice 2 dominates 3 $B = \begin{pmatrix} 2 & 1 & 0 \\ 3 & 2 & 2 \\ 0 & -1 & 1 \end{pmatrix}$ Choice 1 dominates choice 2.

$\therefore$ Reduces to $\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ & $\begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix}$ dominates.

Therefore Nash equilibrium is choice 2 & 1 with payoffs of 2 & 3.

③ $u_1(x, y) = y^2 - x^2 - 2x + y$
 $\frac{\partial u_1}{\partial x} = -2x - 2 = 0 \Rightarrow x = -1$
 $\frac{\partial^2 u_1}{\partial x^2} = -2 < 0$

$u_2(x, y) = 2x^2 - xy - 3y^2 - 3x + 4y$
 $\frac{\partial u_2}{\partial y} = -x - 6y + 4 = 0$
 $6y = 4 - x = 5 \Rightarrow y = 5/6$
 $\frac{\partial^2 u_2}{\partial y^2} = -6 < 0$

Nash equilibrium $x = -1$ & $y = 5/6$

④ $A = \begin{pmatrix} 0 & 3 \\ 2 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ $p_1 = p$ $p_2 = 1-p$, $q_1 = q$ $q_2 = 1-q$
 $u_1 = 3p(1-q) + 2(1-p)q$
 $\frac{\partial u_1}{\partial p} = 3(1-q) - 2q = 3 - 5q$ $q = 3/5$ $u_2 = 2pq + 1(1-p)(1-q)$
 $\frac{\partial u_2}{\partial q} = 2p - (1-p) = 3p - 1$ $p = 1/3$

Mixed strategy 1st player $\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix}$; 2nd player $\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} 3/5 \\ 2/5 \end{pmatrix}$.