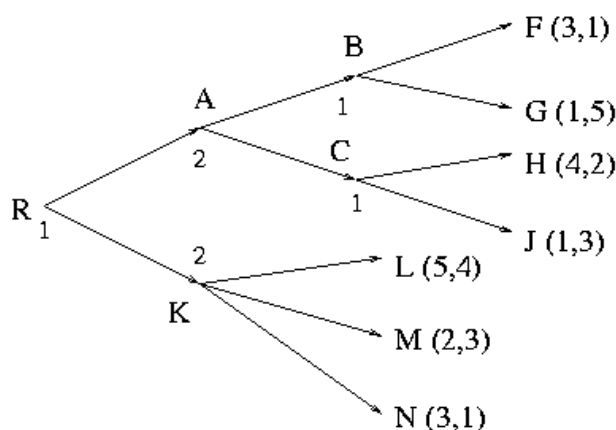


1. (25 Points) Consider the bimatrix game where the payoff for player one is given by the matrix A and the payoff for player two is given by the matrix B :

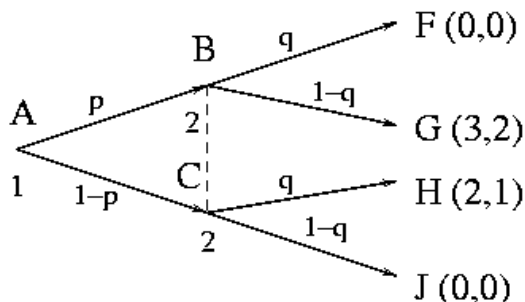
$$A = \begin{pmatrix} 1 & 1 & 4 \\ 4 & 0 & 7 \\ 2 & 3 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 4 & 7 & 2 \\ 0 & 6 & 7 \\ 1 & 5 & 3 \end{pmatrix}.$$

Find all the (pure) Nash equilibrium.

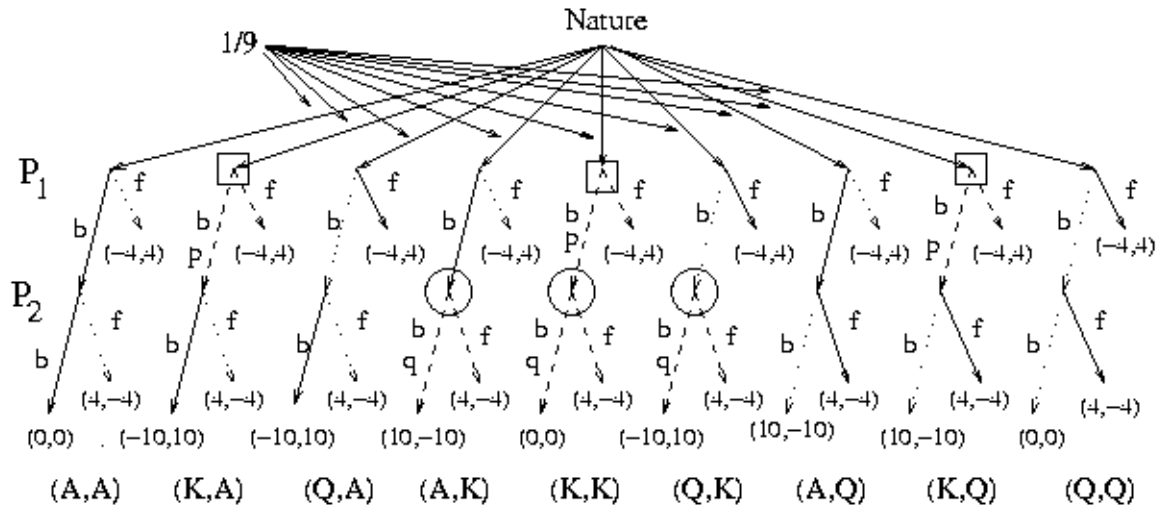
2. Consider the sequential game with perfect information given in the following game tree.
- (25 Points) Find a subgame perfect equilibrium using pure strategies. Give the decision at each node.
 - (25 Points) Give a Nash equilibrium for the game that is not a subgame perfect equilibrium. Explain why your answer is correct.



3. (25 Points) Consider the sequential game with imperfect information given in the following game tree. Player one owns node A. Player two owns the information set $\{B, C\}$. Let p be the behavior strategy for player 1 on edge AB, and let q be the behavior strategy for player 2 on the edges BF and CH. Using Bayesian consistent system of beliefs, find the values of p and q that gives the sequential equilibrium. What is the system of beliefs for player 2 on the information set $\{B, C\}$?



4. (25 Points) Consider the game with imperfect information where “nature” deals hands to players 1 and 2. Each player receives one card, which has probability $\frac{1}{3}$ of being an ace, A, probability $\frac{1}{3}$ of being a king, K, and probability $\frac{1}{3}$ of being a queen, Q. Thus the probability of each of the nine possible hands is $\frac{1}{9}$. Each player knows his or her own card but not the card of the other player. Assume the ante is 4 before the games starts. Each player in turn can fold, f, or bet 6 more, b, to stay in the game. See the following game tree. Assume each player always bets if they have an ace. Also assume each player always folds if they have a queen. (The edges where the choices are fixed are shown with solid lines and the alternative choices never taken are shown as dotted lines. The edges where the choices are involved are shown as dashed lines.) Let p be the behavior strategy for player 1 to bet with a king, and q the behavior strategy for player 2 to bet with a king.
- What is the expected payoff in terms of p and q for player 1 on the information set where he holds a king? (The nodes in this information set have small squares around them in the game tree below.)
 - What is the consistent system of beliefs for player 2 on the hands where she holds a king and needs to bid or fold? Give the answer in terms of p . (The nodes in this information set have small circles around them in the game tree below.)
 - What is the expected payoff in terms of p and q for player 2 on the hands where she holds a king and needs to bid or fold? (The nodes in this information set have small circles around them.)
 - What choice of q , $0 \leq q \leq 1$, maximizes the payoff for player 2 on the hands where she holds a king and needs to bid or fold?
 - What choice of p , $0 \leq p \leq 1$, maximizes the payoff for player 1 the hands where he holds a king and needs to bid or fold?



5. Consider the bargaining problem with the feasible allocation set

$$s_1 + s_2 \leq 4, \quad 0 \leq s_1 \leq \frac{16}{9}, \quad 0 \leq s_2,$$

with disagreement values of $s_1 = s_2 = 0$. Suppose the utility functions are $u_1(s_1, s_2) = \sqrt{s_1}$ and $u_2(s_1, s_2) = s_2$.

- (a) (25 Points) What is the Nash solution of the bargaining? What are the values of the s_j 's and the u_j 's at the solution?
- (b) (25 Points) What is the Kalai-Smorodinsky solution of the bargaining? What are the values of the s_j 's and the u_j 's at the solution?
6. (25 Points) Consider the sequential bargaining situation in the following figure. (In the figure $P1$ is player one, $P2$ is player two. $s1 = s_1$ is the offer of the amount player one would receive at time $t = 1$, $s2 = s_2$ is the offer of the amount player one would receive at time $t = 2$, accepting the offer is labeled A , and rejecting the offer is labeled R .) Assume the discount factor $\delta_1 = \delta_2 = 0.9$ for both players. Assume the continuation payoff at time $t = 3$ is $(x, 1 - x)$, with $0 \leq x \leq 1$. Derive the choices of both players at times $t = 2$ and $t = 1$. Indicate why these are the correct choices for each player for any x as indicated. Finally, what are the payoffs at $t = 1$ if $x = 0$?

