

ANSWERS TO TEST 1, APRIL 17 2001

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(1) For the bonds, the money at the end will be $(1.08) 1000 = 1080$. For the stocks, the money at the end will be (i) $(0.9) 1000 = 900$ with probability 0.2, (ii) $(1.05) 1000 = 1050$ with probability 0.4, and (iii) $(1.2) 1000 = 1200$ with probability 0.4. Both have an expected final amount of money of \$1080, $0.2(900) + 0.4(1050) + 0.4(1200) = 1080$.

(1a) Assume that $u(w) = w^2$. The utility of the bonds is $u(1080) = 1080^2 = 1166400$. The utility of the stocks is $0.2(900^2) + 0.4(1050^2) + 0.4(1200^2) = 1179000$. The utility of the stocks is higher. This person, with a risk seeking utility, prefers the stocks with the potential for higher return to the certainty of the return of the bonds.

(1b) Assume that $u(w) = \sqrt{w}$. The utility of the bonds is $u(1080) = \sqrt{1080} = 32.86$. The utility of the stocks is $0.2\sqrt{900^2} + 0.4\sqrt{1050^2} + 0.4\sqrt{1200^2} = 32.82$. The utility of the bonds is higher. This person, with a risk averse utility, prefers the bonds with the certainty of the return to the stocks with the potential for either higher or lower return.

(2) For the row player, if the first column is chosen then maximum is at the third row, $a_{31} = 3$, if the second column is chosen then maximum is at the first row, $a_{12} = 5$, and if the third column is chosen then maximum is at the first row, $a_{13} = 3$. For the column player, if the first row is chosen then the maximum is at the second column, $b_{12} = 3$, if the second row is chosen then the maximum is at the second column, $b_{22} = 1$, and if the third row is chosen then the maximum is at the first column, $b_{31} = 4$. There are two choices in common, either the first row and second column, or the third row and first column. Both of these are Nash equilibria.

(3) Let p_1, p_2 , and $p_3 = 1 - p_1 - p_2$ be the probabilities of playing the first, second, and third rows. Let q_1, q_2 , and $q_3 = 1 - q_1 - q_2$ be the probabilities of playing the first, second, and third columns. The pay offs are

$$\begin{aligned}\pi_1 &= p_1 2q_2 + p_2[q_1 + (1 - q_1 - q_2)] + (1 - p_1 - p_2) 3q_1 \\ \pi_2 &= q_1 2p_2 + q_2[p_2 + (1 - p_1 - p_2)] + (1 - q_1 - q_2) p_1.\end{aligned}$$

Taking the partial derivatives of π_1 with respect to p_1 and p_2 and those of π_2 with respect to q_1 and q_2 we get

$$\begin{aligned}\frac{\partial \pi_1}{\partial p_1} &= 2q_2 - 3q_1 \\ \frac{\partial \pi_1}{\partial p_2} &= 1 - q_2 - 3q_1 \\ \frac{\partial \pi_2}{\partial q_1} &= 2p_2 - p_1 \\ \frac{\partial \pi_2}{\partial q_2} &= 1 - p_1 - p_1.\end{aligned}$$

Setting these equal to zero, the first equation gives $q_2 = \frac{3}{2}q_1$. Substituting in the second equation, gives $1 = \frac{3}{2}q_1 + 3q_1 = \frac{9}{2}q_1$, or $q_1^* = \frac{2}{9}$. Then, $q_2^* = \frac{3}{2} \cdot \frac{2}{9} = \frac{3}{9}$, and $q_3^* = 1 - q_1^* - q_2^* = \frac{4}{9}$. The fourth equation gives $2p_1 = 1$, or $p_1^* = \frac{1}{2}$. The third equation gives $p_2^* = \frac{1}{2}p_1^* = \frac{1}{4}$. Finally, $p_3^* = 1 - p_1^* - p_2^* = \frac{1}{4}$. This gives the Nash equilibrium with mixed strategies

$$(p_1^*, p_2^*, p_3^*) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right) \quad (q_1^*, q_2^*, q_3^*) = \left(\frac{2}{9}, \frac{3}{9}, \frac{4}{9}\right).$$

(4) The partial derivatives of u_1 with respect to x and those of u_2 with respect to y are

$$\begin{aligned}\frac{\partial u_1}{\partial x} &= -y - 2x + 2 \\ \frac{\partial u_2}{\partial y} &= -2x - 6y + 5.\end{aligned}$$

Setting these equal to zero gives the following two equations

$$\begin{aligned}2 &= 2x + y \\ 5 &= 2x + 6y.\end{aligned}$$

Subtracting the first equation from the second gives $3 = 5y$ or $y^* = \frac{3}{5}$. Then, $2x = 2 - \frac{3}{5} = \frac{7}{5}$ or $x^* = \frac{7}{10}$.

The first is strictly negative on $[0, 1]$, so u_1 has a maximum at the Thus, the Nash equilibrium is at $x^* = \frac{7}{10}$ and $y^* = \frac{3}{5}$.

(5) We analyze the problem by backward induction. The node B is only followed by terminal nodes, so its cost is 1 for BD . Node A can either go to B with a cost of $2 + 1 = 3$ or to D with a cost of 4: the minimum is AB with a cost of 3. Node C can either go to B with a cost of $2 + 1 = 3$ or to D with a cost of 5: the minimum is CB with a cost of 3. Finally, node R can either go to A with a cost of $1 + 3 = 4$, or to B with a cost of $4 + 1 = 5$, or to C with a cost of $2 + 3 = 5$: the minimum is RA with a cost of 4. The path is $RABD$.