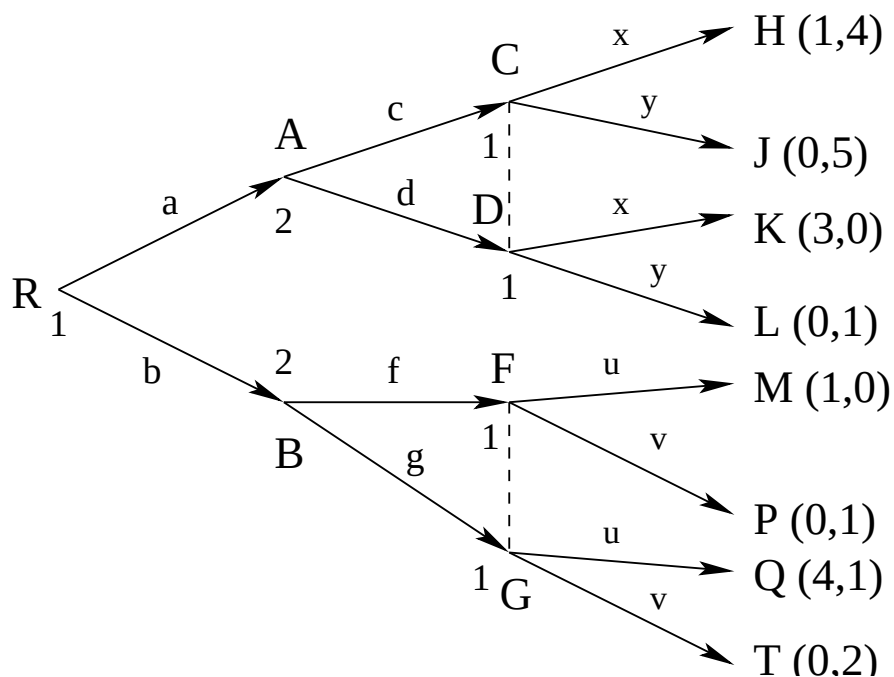


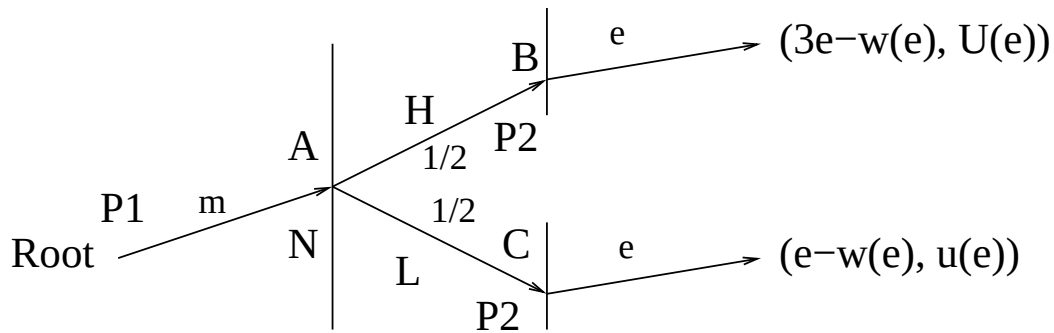
Start each problem on a new page.

Calculators are permitted.

1. (20 Points) Consider the sequential game with imperfect information given in the following game tree. Nodes $\{R, C, D, F, G\}$ are owned by player one and nodes $\{A, B\}$ are owned by player two. The nodes $\{C, D\}$ form one information set, and nodes $\{F, G\}$ form a second information set. Show that the (pure) strategy $\{b, x, u\}$, $\{d, g\}$ is a Nash equilibrium but not a subgame perfect equilibrium.

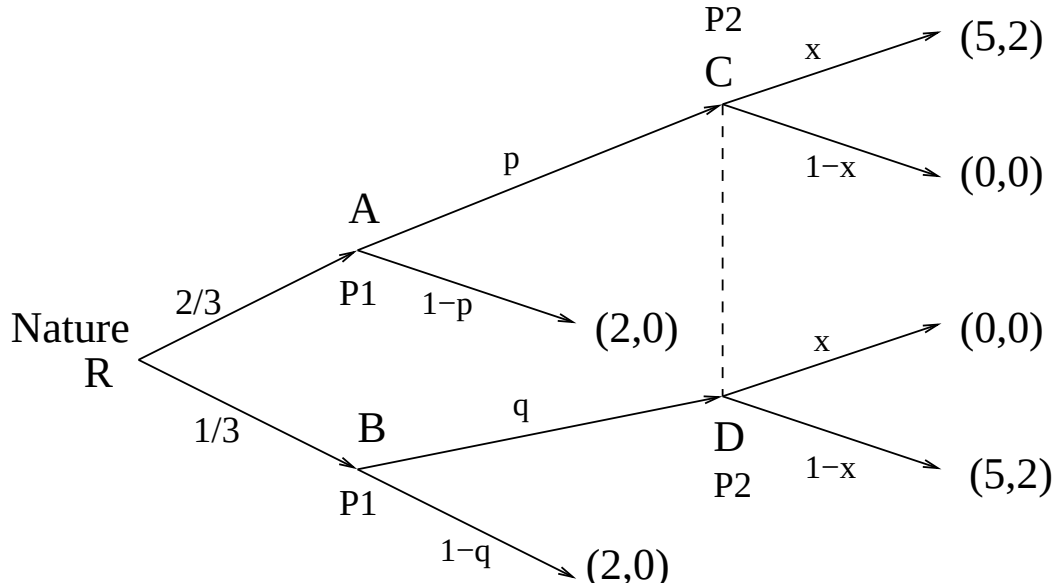


2. (30 Points) Consider the job market signaling game where the firm (player one, $P1$) pays a wage of $w(e) = \frac{1}{4}e^2 + me + 0.1$ based on the education level of e of the individual (player two, $P2$). The pool of individuals is $1/2$ of high ability H and $1/2$ of low ability L . The payoff to an individual with high ability with education level $e \geq 0$ is $U(e) = w(e) - \frac{1}{2}e^2$; the profit for the firm is $3e - w(e)$ if the individual is high ability. The payoff to an individual with low ability with education level $e \geq 0$ is $u(e) = w(e) - \frac{3}{4}e^2$; the profit for the firm is $e - w(e)$ if the individual is low ability. The firm picks the variable $m \geq 0$ and the individual picks the variable $e \geq 0$. See the game tree.



- What is the choice of an education level $e_H(m)$ by the individual at node B with high ability which maximize the individual's payoff?
- What is the choice of an education level $e_L(m)$ by the individual at node C with low ability which maximize the individual's payoff?
- What is the expected payoff of the firm at the Root based on the choice of m and given the choices of the education level by individual at nodes B and C ? What choice of m maximizes the payoff for the firm?

3. (50 Points) Consider the sequential game with imperfect information given in the following game tree. The root R is owned by nature; there is a probability of $2/3$ of taking edge RA and a probability of $1/3$ of taking edge RB . Player one owns nodes A and B . Player 2 owns the information set $\{C, D\}$. The behavioral strategies are as given in the game tree with probabilities, p , $1 - p$, q , $1 - q$, x , and $1 - x$.



- What is the Bayesian consistent system of beliefs for player two on the information set $\{C, D\}$? Give the answer in terms of the variables for the behavioral strategies of the players.
- What is the expected payoff for player two on the information set $\{C, D\}$? Give the answer in terms of the variables for the behavioral strategies of the players. What choice of x should player two make to maximize the payoff? Give the answer in terms of the variables p and q for the strategies of player one.
- What is the expected payoff for player one at the node A ? What choices of p should player one make to maximize the payoff? Give the answer in terms of the variable x for the strategy of player two.
- What is the expected payoff for player one at the node B ? What choice of q should player one make to maximize the payoff? Give the answer in terms of the variable x for the strategy of player two.
- Split up the values of x into various intervals and values based on the answers two the two previous part. On these intervals, what are the resulting values of p and q ? Do they give compatible choices with the choice of x given in part (b)? What are the possible sequential equilibrium for this sequential game with imperfect information?