

No books, no notes

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1. (30 Points) Consider the game tree with perfect information given in Figure 1.
 - a. What is the subgame perfect Nash equilibrium in pure strategies? Give the complete strategy profile.
 - b. Find a strategy profile that is a Nash equilibrium but not subgame perfect.

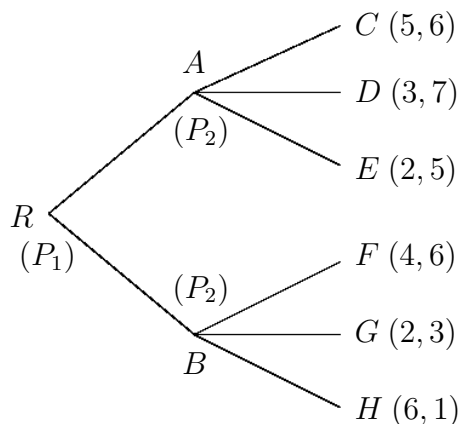


FIGURE 1

2. (30 Points) Consider the game given in Figure 2. Nature owns the root and has a $\frac{1}{2}$ probability of selecting the edge to node A and $\frac{1}{2}$ probability of selecting the edge to node B . The other behavior strategy variables are given in the figure.
 - a. For $(p, q) \neq (0, 0)$, what is the consistent system of beliefs for P_2 on the information set $\mathcal{I} = \{C, D\}$?
 - b. If $(p, q) = (0, 0)$, show that any system of beliefs on $\mathcal{I}$ is consistent.
 - c. For $(p, q) \neq (0, 0)$, what is the expected payoff for P_2 on $\mathcal{I}$? What choice of x maximizes the expected payoff? Include the case $(p, q) = (0, 0)$ and give the answer in terms of various cases of p and q .

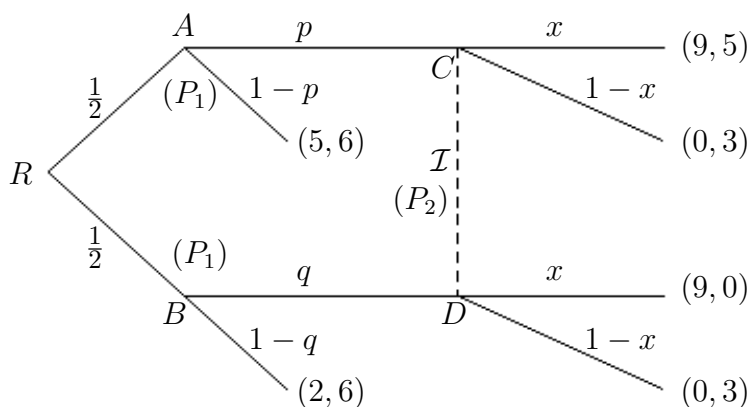


FIGURE 2

3. (40 Points) Consider the sequential game with imperfect information given in Figure 3. Assume that the derivative of the expected values of the payoffs using the system of consistent beliefs on the information sets $\mathcal{I}_1 = \{X, Y\}$, $\mathcal{I}_2 = \{Z, F\}$, P_1 , and $\mathcal{I}_3 = \{G, H\}$ and node R as follows.

$$\begin{aligned}\frac{\partial E_1(R, \sigma)}{\partial x} &= p(5q - 1) - r(4q - 2) - 2, \\ \frac{\partial E_2(\mathcal{I}_1, \sigma, \mu^\sigma)}{\partial q} &= 1 - 6r - x(4 + 2p - 6r) \equiv \Delta, \\ \frac{\partial E_1(\mathcal{I}_2, \sigma, \mu^\sigma)}{\partial p} &= -1 + 5q, \\ \frac{\partial E_1(\mathcal{I}_3, \sigma, \mu^\sigma)}{\partial r} &= 4q - 2,\end{aligned}$$

Note: The systems of beliefs on the information sets are completely determined by the behavior strategy in this game, so you do not need to worry about the beliefs being arbitrary.

- (25 Points) Find the sequential equilibrium with $q = 1/2$. Hint: Solve for the other behavior strategies in the following order: p , x , and then r .
- (15 Points) Why is $q = 1/2$, $r = 1/6$, $x = 0$, and $p = 0$ a Nash equilibrium in behavior strategies but **NOT** a sequential equilibrium? Is this second solution subgame perfect?

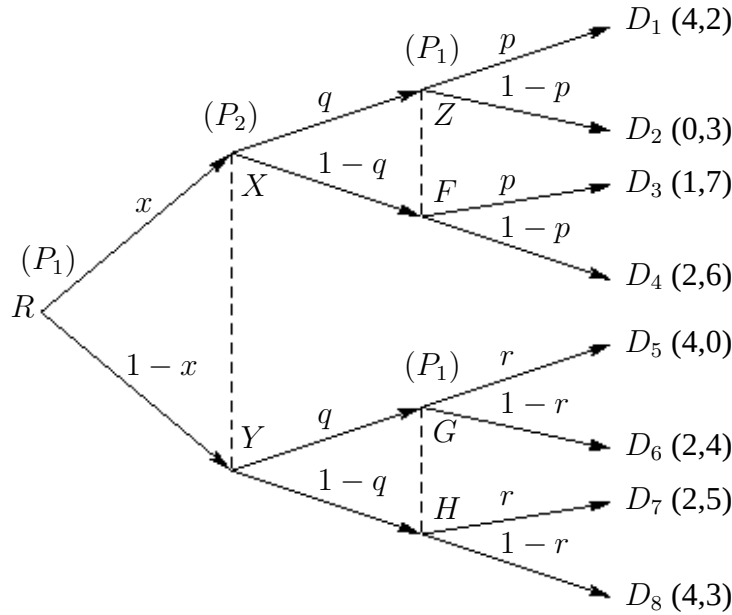


FIGURE 3