

No books, no notes

Show all your work in your bluebook. Start each problem on a new page.

1. (16 Points) For a collection of lotteries, there are three possible outcomes, a_1 , a_2 , and a_3 . The payoff function satisfies $u(a_1) = 0$, $u(a_2) = v$, and $u(a_3) = 10$. A lottery (p_1, p_2, p_3) indicates a lottery in which the probability of getting outcome a_i is p_i . The decision maker prefers the lottery $(0.1, 0.4, 0.5)$ to $(0.3, 0, 0.7)$ to $(0.275, 0.125, 0.6)$. Are these preferences consistent? And, what can you say about the value of $v = u(a_2)$? Note: $0.125 = 1/8$.
2. (16 Points) (Cournot's duopoly) Because of the form of the costs and inverse demand function, the profit of two firms is given by

$$\pi_1(q_1, q_2) = q_1(10 - 2q_2) - 2q_1^2 \quad \text{and} \quad \pi_2(q_1, q_2) = q_2(8 - 2q_1) - q_2^2,$$
 for production levels $q_1 \geq 0$ by the first firm and $q_2 \geq 0$ by the second firm. Find the Nash equilibrium.
3. Consider the game tree with perfect information given in Figure 1.
 - a. (14 Points) What is the subgame perfect Nash equilibrium? Give the strategy profile and payoffs for the players.
 - b. (14 Points) Find a strategy profile that is a Nash equilibrium but not subgame perfect. Explain why your choice (i) is not subgame perfect, and (ii) is a Nash equilibrium.

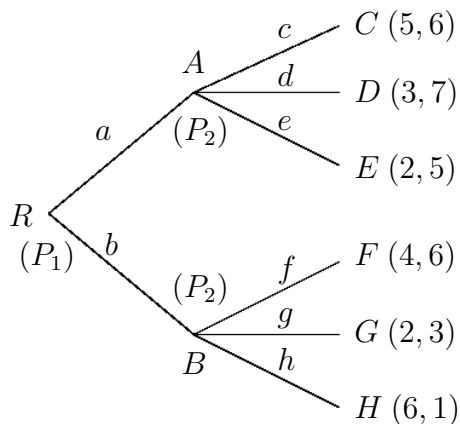


FIGURE 1

4. (16 Points) In a game of attrition, the payoff functions of two players are given by

$$u_1(t_1, t_2) = \begin{cases} -t_1 & \text{if } t_1 < t_2 \\ 5 - t_2 & \text{if } t_1 = t_2 \\ 10 - t_2 & \text{if } t_1 > t_2 \end{cases} \quad \text{and} \quad u_2(t_1, t_2) = \begin{cases} -t_2 & \text{if } t_2 < t_1 \\ 6 - t_1 & \text{if } t_2 = t_1 \\ 12 - t_1 & \text{if } t_2 > t_1, \end{cases}$$

where $t_1 \geq 0$ is the time player one waits and $t_2 \geq 0$ is the time player two waits. What is the best response function of player one, $B_1(t_2)$.

Hint: Consider the three cases: (i) $t_2 < 10$, (ii) $t_2 = 10$, and (iii) $t_2 > 10$.

(over for problem 5)

5. Consider the bi-matrix game given by

$$\begin{pmatrix} (0, 0) & (8, -4) & (8, -4) \\ (2, -2) & (0, 0) & (2, -2) \\ (1, -1) & (1, -1) & (0, 0) \end{pmatrix}.$$

- a.** (8 Points) Are there any pure strategy Nash equilibria? Explain by giving the best response in pure strategies of each player to the pure strategies of the other player.
- b.** (16 Points) Consider mixed strategies, where player one (who chooses the row) has a strategy (p_1, p_2, p_3) and player two (who chooses the column) has a strategy (q_1, q_2, q_3) .
- (i) For $p_3 = 0 = q_3$ and $1 \leq i \leq 3$, find the expected payoffs $E_1(r_i, (q_1, q_2, 0))$ for rows r_i and $E_2((p_1, p_2, 0), c_i)$ for columns c_i .
- (ii) Find a mixed strategy Nash equilibrium with $p_3 = 0 = q_3$. Indicate why it is a Nash equilibrium and determine the payoff for each player.