

Math 364: Supplemental Problems

§1.1 Extrema of functions

- Find the maximum and minimum of each of the following functions on the interval $0 \leq x \leq 3$.
 - $f(x) = x^2 - 2x + 2$.
 - $g(x) = -x^2 + 2x + 4$.
 - The function $h(x)$ satisfies $h'(x) > 0$ for all $0 \leq x \leq 3$. Find the points at which the function attains its maximum and minimum, not the values.
 - The function $k(x)$ satisfies $k'(x) < 0$ for all $0 \leq x \leq 3$. Find the points at which the function attains its maximum and minimum, not the values.
 - The function $u(x)$ satisfies $u'(x) = 0$ for all $0 \leq x \leq 3$. Find the points at which the function attains its maximum and minimum, not the values.

§2.8 Best responses

- Consider the strategic game with payoffs given by the following bi-matrix:

	L_2	M_2	R_2
H_1	(3, 2)	(1, 0)	(0, 1)
M_1	(2, 0)	(1, 1)	(3, 2)
L_1	(0, 3)	(2, 3)	(3, 1)

- Give the best response functions for both players. (Use pure actions as given in Chapter 2, not mixed strategies as in Chapter 4.)
- Give all the Nash equilibria.

§4.1 Valuations of a Lottery

- You are a individual whose net worth is \$10,000 and you are thinking of opening a Donut Franchise. To open the franchise you must invest \$5,000. If you buy the franchise, the probability is p_1 that you will lose the entire investment, p_2 that you will break even, and p_3 that you will make \$500,000. Thus, your net worth would be \$5,000, \$10,000, and \$510,000 in the three cases.
 - If you are risk-neutral so you evaluate the payoffs using the net worth, what would be your decision if $p_1 = p_2 = p_3 = 1/3$? Hint: Is your expected payoff greater than \$10,000?
 - If you are risk-neutral, $p_1 = p_2 = x$, and $p_3 = 1 - 2x$, for which values of x and $1 - 2x$ would you buy the franchise? Hint: For which values of x and $1 - 2x$ is your expected payoff greater than \$10,000?
 - If you use the function $g(w) = \ln(w)$ to rescale your payoffs, $p_1 = p_2 = x$, and $p_3 = 1 - 2x$, for which values of x and $1 - 2x$ would you buy the franchise? Hint: For which values of x and $1 - 2x$ is the expected payoff rescaled by $\ln(w)$ greater than $\ln(10000)$?

§4.10 Mixed Strategy Nash Equilibria

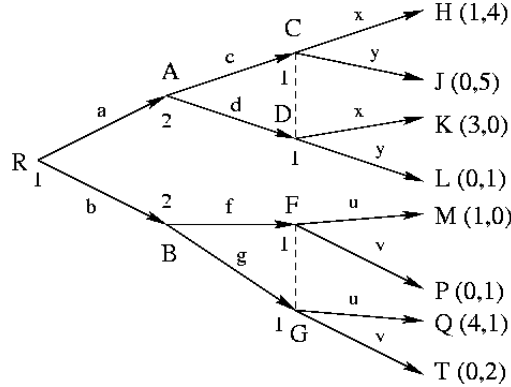
- Find all the (pure and mixed strategy) Nash equilibrium for the bi-matrix game

$$\begin{pmatrix} (0, 4) & (2, 0) & (0, 0) \\ (1, 0) & (0, 1) & (0, 0) \\ (0, 0) & (0, 0) & (1, 2) \end{pmatrix}.$$

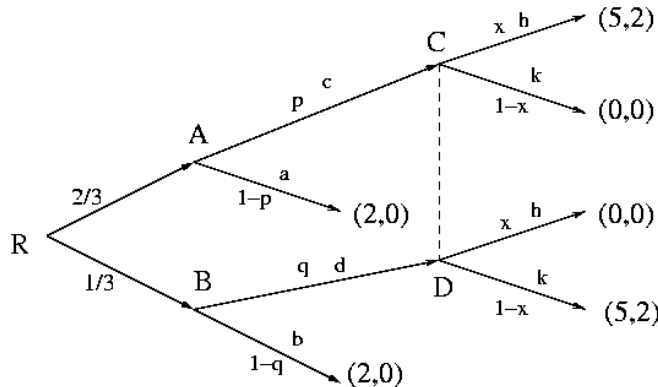
Hint: Use either (i) the method of the book or (ii) the substitutions $p_1 = 1 - p_2 - p_3$ and $q_1 = 1 - q_2 - q_3$.

§10.4 Sequential Equilibria

- Consider the sequential game with imperfect information given in the following game tree. Nodes $\{R, C, D, F, G\}$ are owned by player P_1 and nodes $\{A, B\}$ are owned by player P_2 . There are two information sets, $\mathcal{I}_1 = \{C, D\}$ and $\mathcal{I}_2 = \{F, G\}$. Find a Nash equilibrium in (pure) strategies.



- Consider the extensive game with imperfect information given in the following game tree. The root R is owned by chance; there is a probability of $2/3$ of taking edge RA and a probability of $1/3$ of taking edge RB . Player P_1 owns nodes A and B , while P_2 owns the information set $\mathcal{I} = \{C, D\}$. The behavioral strategies are $\beta_1(c) = p$, $\beta_1(a) = 1 - p$, $\beta_1(d) = q$, $\beta_1(b) = 1 - q$, $\beta_2(h) = x$ and $\beta_2(k) = 1 - x$.



- What is the weakly consistent belief system for P_2 on the information set $\mathcal{I}$? Give the answer in terms of the variables for the behavioral strategies of the players.
- What is the expected payoffs for P_2 choosing h and k on $\mathcal{I}$? Give the answer in terms of the variables for the behavioral strategies of the players. What choice of x should player P_2 make to maximize the payoff on $\mathcal{I}$? Give the answer in terms of cases involving the values of p and q .
- What is the expected payoff for P_1 choosing c and a at the node A ? What choices of p should player P_1 make to maximize the payoff? Give the answer in terms of the variable x for the strategy of player P_2 .

- d. What is the expected payoff for P_1 choosing d and b at the node B ? What choice of q should player P_1 make to maximize the payoff? Give the answer in terms of the variable x for the strategy of player P_2 .
- e. Split up the values of x into various intervals and values based on the answers to the two previous parts. On these intervals, what are the resulting values of p and q ? Do they give compatible choices with the choice of x given in part (b)? What are the possible sequential equilibrium for this extensive game with imperfect information?

§10.5 Signaling

1. (cf. 342.1)

Consider the educational signaling game given in Figure 341.1 (or the refinement in class). Assume $0 < \pi < 1$. Find a pooling sequential equilibrium for which $e_H = e_L$ and the payoff for the worker is optimal, i.e., the payoff for the work is optimal over e_H , e_L , and the choice of the firm. (This is a slight modification of the discussion in the book. There is then only one value of $e_H = e_L$ and not a range.) What is the assessment for this sequential equilibrium? Verify that this assessment is a sequential equilibrium.

2. (Refinement of 335.1)

Consider the game that differs from the one in Figure 333.1 only in the payoffs to the terminal histories (r_w, A_r) and (r_w, F_r) : $a_i = u_i(r_w, A_r)$ and $b_i = u_i(r_w, F_r)$ for $i = 1, 2$.

- a. What conditions on a_1 , a_2 , b_1 , and b_2 allow a separating sequential equilibrium with $\beta(r_s) = 1 = \beta(u_w)$? What is the assessment for this sequential equilibrium? Verify that this assessment is a sequential equilibrium.
- b. For $0 < p < 1/4$, what conditions on a_1 , a_2 , b_1 , and b_2 allow a pooling sequential equilibrium with $\beta(u_s) = 1 = \beta(u_w)$, $\beta(F_r) = 1 = \beta(F_u)$? What is the assessment for this sequential equilibrium? Verify that this assessment is a sequential equilibrium.

§13.2 Evolutionary Equilibria

1. Consider the evolutionary strategic game whose payoff matrix is given by $\begin{bmatrix} 2 & 0 \\ 3 & -1 \end{bmatrix}$. Find all the Nash equilibria and check if any of them is an ESS.