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- (16 Points) For a collection of lotteries, there are three possible outcomes, a_1 , a_2 , and a_3 . The payoff function satisfies $u(a_1) = 0$, $u(a_2) = v$, and $u(a_3) = 10$. A lottery (p_1, p_2, p_3) indicates a lottery in which the probability of getting outcome a_i is p_i . The decision maker prefers the lottery $(0.1, 0.5, 0.4)$ to $(0.3, 0, 0.7)$ to $(0.2, 0.2, 0.6)$. What choice if any for the value of v make these preferences induced by the Bernoulli payoff function induced from u ?
- (17 Points) Assume that two countries that impose a tariff on a product of $x \geq 0$ by country one and $y \geq 0$ by country two, and that the payoffs are $u_1(x, y) = 2000 + 60x + xy - x^2 - 90y$ and $u_2(x, y) = 2000 + 60y + xy - y^2 - 90x$. Find the Nash equilibrium.
- (17 Points) In one form of the game of attrition, the payoff functions of two players are given by

$$u_1(t_1, t_2) = \begin{cases} 0 & \text{if } t_1 < t_2 \\ \frac{1}{2}(v_1 - t_2) & \text{if } t_1 = t_2 \\ v_1 - t_2 & \text{if } t_1 > t_2 \end{cases} \quad \text{and} \quad u_2(t_1, t_2) = \begin{cases} 0 & \text{if } t_2 < t_1 \\ \frac{1}{2}(v_2 - t_1) & \text{if } t_2 = t_1 \\ v_2 - t_1 & \text{if } t_2 > t_1, \end{cases}$$

where $t_1 \geq 0$ is the time player one waits and $t_2 \geq 0$ is the time player two waits. Assume that $0 < v_2 < v_1$. Give the best response function of each player, $B_1(t_2)$ and $B_2(t_1)$.

- (25 Points) Consider the bi-matrix game given by

$$\begin{pmatrix} (8, 1) & (0, 1) & (2, 3) \\ (2, 1) & (4, 4) & (0, 0) \\ (1, 3) & (3, 0) & (3, 2) \end{pmatrix}.$$

- Are there any pure strategy Nash equilibria? Explain by giving the best response in pure strategies of each player to the pure strategies of the other player.
 - Consider mixed strategies, where player one (who chooses the row) has a strategy (p_1, p_2, p_3) and player two (who chooses the column) has a strategy (q_1, q_2, q_3) .
 - For $p_2 = 0 = q_2$ and $1 \leq i \leq 3$, find the expected payoffs $E_1(r_i, (q, 0, 1 - q))$ for rows r_i and $E_2((p, 0, 1 - p), c_j)$ for columns c_j .
 - Find a mixed strategy Nash equilibrium with $p_2 = 0 = q_2$. Indicate why it is a Nash equilibrium.
- (25 Points) For an evolutionary game, members of a single population are randomly matched in pairs and have the following payoff matrix of playing row r_i against column c_j ,

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}.$$

- Find all the (symmetric) Nash equilibria.
- Check whether each Nash equilibrium is an ESS.