Errata for
Introduction to Dynamical Systems: Discrete and Continuous, 1st edition

by R. Clark Robinson
Published by Pearson Prentice Hall, 2004

p. 9 L. 24: $\sin(x_1)$ should be $\sin(x)$.

p. 14 L. 15: Equation 2.0.4 should refer to the un-numbered first equation of the chapter.

p. 15 L. 4: This equation should have $\mathbf{A}(t)$.

p. 15 L. -8: If $\mathbf{M}(t)$ is a matrix solution and we write $\mathbf{c}$ for the constant vector with entries c_j, then

$$\frac{d}{dt}\mathbf{M}(t)\mathbf{c} = \mathbf{A}(t)\,\mathbf{M}(t)\mathbf{c},$$

and so $\mathbf{M}(t)\mathbf{c} = c_1\,\mathbf{x}^1(t) + \cdots + c_n\,\mathbf{x}^n(t)$ is the vector solution that equals

$$\mathbf{M}(0)\mathbf{c} = c_1\,\mathbf{x}^1(0) + \cdots + c_n\,\mathbf{x}^n(0)$$

when $t = 0$.

p. 16 Theorem 2.1.1(c): If $\mathbf{c} = (c_1, \ldots, c_k)^{\mathsf{T}}$ is a constant vector and $\mathbf{M}(t)$ is an $n \times k$ matrix solution, then $\mathbf{M}(t)\mathbf{c}$ is a (vector) solution.

p. 16 L. 23: is called the *Wronskian* of the system of vector solutions given by the columns of $\mathbf{M}(t)$.

p. 21 L. 19 & 21: $\lambda^2 - (a+d)\lambda + (ad-bc)$ and $\Delta = (ad-bc) = \det(A)$

p. 48 L. -1:

$$\begin{pmatrix} \dot{C}_1 \\ \dot{C}_2 \end{pmatrix} = k \begin{pmatrix} -(1+b) & b \\ (1+b) & -(1+b) \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} + k \begin{pmatrix} C_0(t) \\ 0 \end{pmatrix}.$$

p. 49 L. 1: Since $(1+b)^2 = 1 + 2b + b^2 > b + b^2$

p. 54 L. -12: Therefore,

$$\frac{d}{dt}\|\mathbf{x}(t)\|^2 \le 2C\|\mathbf{x}(t)\|^2$$

for some constant C. In this case, by dividing by $\|\mathbf{x}(t)\|^2$ and integrating, we see that

$$\|\mathbf{x}(t)\|^2 \le \|\mathbf{x}(0)\|^2 e^{2Ct} \qquad \text{or}$$
$$\|\mathbf{x}(t)\| \le \|\mathbf{x}(0)\| e^{Ct}.$$

(This is a special case of Gronwall's inequality given in Lemma 3.3.4, which covers the case when $\mathbf{x}(t) = \mathbf{0}$ for some t.)

p. 64 Ex. 2.1.5(b): $e^{t\mathbf{A}}\,e^{t\mathbf{A}}$ should be $e^{t\mathbf{A}}\,e^{t\mathbf{B}}$.

p. 65 Ex. 2.2.4: The equation should be $m\,\ddot{y} + b\,\dot{y} + k\,y = 0$.

p. 70 L -3:

$$t^{+}_{x_0} = \frac{1}{r}\Big(\ln(K - x_0) - \ln(|x_0|)\Big).$$

p. 87 L 12: $\tau < \min\left\{ \dfrac{r}{K}, \dfrac{1}{L} \right\}$

p. 89 L. 12: Insert the following sentence: "For a small time interval, both solutions are in some closed ball $\bar{\mathbf{B}}(\mathbf{x}_0, r)$ and there is some constant L as in Theorem 3.3.1." Then,

p. 111 L -11: "next chapter" should be "Chapter 6"

p. 118 L 11: "the the" should be "the"

p. 125 Fig. 4.5.2: The labels L and ${}^{K}\!/_{a}$ along the y-axis should be interchanged.

p. 126 L 9:

$$DF_{(\mathbf{x}^*)} = \frac{1}{1+\alpha+\beta}\begin{pmatrix} -1 & -\alpha & -\beta \\ -\beta & -1 & -\alpha \\ -\alpha & -\beta & -1 \end{pmatrix}.$$

p. 126 L 11: $\lambda_2 = \bar{\lambda}_3 = \dfrac{1}{1+\alpha+\beta}\left(-1 - \alpha\, e^{i2\pi/3} - \beta\, e^{i4\pi/3}\right),$

p. 126 L 13:

$$\frac{1}{1+\alpha+\beta}\left(-1 + \frac{\alpha+\beta}{2}\right) = \frac{\alpha+\beta-2}{2(1+\alpha+\beta)} > 0.$$

p. 127 L 1: Therefore, any orbit with $S(0) > 0$ must enter and remain in the set where $S \le 2$.

p. 129 L 2: It should read as follows:

$$\frac{d\,x}{d\,S} = \frac{d}{d\,S}\left(-\frac{D}{\beta\,m}S + \frac{D(S^{(0)}-a)}{\beta\,m} + \frac{D\,a\,S^{(0)}}{\beta\,m\,S}\right)$$

p. 129 L -15:

$$\begin{pmatrix} -D - \dfrac{\beta\,(m-D)\,x^*}{a+S^*} & -\dfrac{\beta\,m\,S^*}{a+S^*} \\ \dfrac{(m-D)\,x^*}{a+S^*} & 0 \end{pmatrix}.$$

p. 129 L -10:

$$\begin{pmatrix} -D & \dfrac{\beta\,m\,S^{(0)}}{a+S^{(0)}} \\ 0 & \dfrac{m\,S^{(0)}}{a+S^{(0)}} - D \end{pmatrix}.$$

Since $\lambda < S^{(0)}$, the eigenvalues are

$$-D < 0 \qquad \text{and} \qquad \frac{m\,S^{(0)}}{a+S^{(0)}} - D > 0,$$

p. 130 L 10: We define …

p. 131 L 7: … Figure 4.6.1. The fixed points are $(x^*, 0)$ and $(0, y^*)$, where

$$x^* = \frac{S^{(0)} - \lambda_1}{\beta_1} \qquad \text{and} \qquad y^* = \frac{S^{(0)} - \lambda_2}{\beta_1}.$$

By phase plane analysis similar to that in Section 4.5 for competitive systems, any trajectory $\phi(t; (x_0, y_0))$ with $x_0 > 0$ and $y_0 > 0$ tends to the fixed point at $(x^*, 0)$ as t goes to infinity.

p. 136 Lemma 4.2.3: In the proof, $\limsup$ and $\liminf$ should be interchanged.

p. 142 Exercise 4.3.1: An easier example to analyze is

$$\begin{aligned} \dot{x} &= y - x^2 \\ \dot{y} &= x - y. \end{aligned}$$

p. 145 Exercise 4.6.1: $\dot{V} = p\mu K + \gamma I - \mu V$.

p. 150 L -8:

$$L^{-1}(C) = \{\,\mathbf{x} : L(\mathbf{x}) = C\,\}.$$

p. 156: In Figures 5.2.7 and 5.2.8, $\pi/2$ should be replaced by π.

p. 157 L -3: "The part with $x_2 \le x \le x_3$ closes up …

p. 164 L 3:

$$\mathbf{U} = \left\{ (x, y) \in \mathbf{U}_1 : 0 \le L(x, y) < \frac{1}{6},\ x > -1 \right\}.$$

p. 164 L 23: (iv) $\dot{L}(\mathbf{x}) < 0$ for $L(\mathbf{x}) = C$.

p. 166 L -11: Insert the following: "Also assume that the differential equation is defined in all of $\bar{\mathbf{B}}(\mathbf{0}, C)$."

p. 167 L 13: $\dot{L} = (-x + 2x^3)y + y\big(x - 2x^3 + y(x^2 - x^4 - y^2)\big)$

p. 167 L -2: only one "See"

p. 174 L -3: "… to generalize to the case …"

p. 175 L 10:

$$\dot{V}(\mathbf{x}) = \sum_i c_i\, f_i^{-1}(f_i(x_i))\, f_i'(x_i)\, \dot{x}_i - \sum_i [\mathbf{f}(\mathbf{x}) + \mathbf{B}]^\top \mathbf{T} \frac{\partial \mathbf{f}}{\partial x_i}(\mathbf{x})\, \dot{x}_i$$

p. 178 Exercise 5.1.2(c): "limit cycles" should be replaced by "periodic orbits"

p. 179 Exercise 5.2.4: $\dot{y} = -x(x-1)(x+2)(x^2-9) = \cdots$

p. 180 Exercise 5.3.4:

$$r_2 = -a_{21}p_1 + a_{22}p_2 + a_{23}p_3$$

p. 181 Exercise 5.3.4(b): Show that

$$\dot{V} = -\sum_{i=1}^{3} c_i a_{ii}(x_i - p_i)^2.$$

p. 193 L 11:

$$h'(x) = \frac{2\,a\ - (2\,b + 1)\,x}{2x^3},$$

p. 196 Figure 6.3.2: The graph of $F(x)$ should be negative for small positive x and positive for small negative x.

p. 199 L -9: Assume the system in polar coordinates $x_1 - x_{\mu,1} = r\,\cos(\theta)$ and $x_2 - x_{\mu,2} = r\,\sin(\theta)$ …

p. 209 L -25 to -17: The constant g is different than the function $g(x, y)$. Also, the formula for $\nabla \cdot (g\,\mathbf{F})_{(x,y)}$ is wrong: Replace with the following:

Consider the equations

$$\begin{aligned}\dot{x} &= x\,(a - b\,y - f\,x)\\ \dot{y} &= y\,(-c + e\,x - h\,y),\end{aligned}$$

where all the parameters a, b, c, e, f, and h are positive. Notice that this is just the predator–prey system, with a negative impact of each population on itself. We assume that $a/f > c/e$, so there is a fixed point (x^*, y^*) in the first quadrant. The first quadrant is invariant, because $\dot{x} = 0$ along $x = 0$ and $\dot{y} = 0$ along $y = 0$. The divergence does not have one sign in the first quadrant:

$$(\nabla \cdot \mathbf{F})_{(x,y)} = a - b\,y - 2\,f\,x - c + e\,x - 2\,h\,y.$$

However, if we let $g(x, y) = 1/xy$, then

$$\nabla \cdot (g\,\mathbf{F})_{(x,y)} = -f/y - h/x,$$

which is strictly negative in the first quadrant.

p. 213 L -14: The iterate P^n is not carefully defined. The following (hopefully) makes it clearer: Continuing by induction, the n^{th} iterate $\mathbf{x}_n = P(\mathbf{x}_{n-1}) = \mathbf{P}^n(\mathbf{x}_0)$ satisfies

$$\mathbf{P}^n(\mathbf{x}_0) - \mathbf{x}^* = \mathbf{P}^n(\mathbf{x}_0) - \mathbf{P}^n(\mathbf{x}^*) = e^{-n\pi/\omega} e^{(\mathbf{A}+\frac{1}{2}\mathbf{I})n2\pi/\omega}\big(\mathbf{x}_0 - \mathbf{x}^*\big).$$

p. 213 L -3: It should be

$$e^{\mathbf{A}2\pi/\omega}\mathbf{x}_0 + \int_0^{\frac{2\pi}{\omega}} e^{\mathbf{A}((2\pi/\omega)-s)} \begin{pmatrix} 0 \\ \cos(\omega\, s + \tau_0) \end{pmatrix} ds$$

p. 214 L 13: The iterate P^n is not carefully defined. Adding the following after the definition of the Poincaré map (hopefully) makes it clearer:
If the map is applied more than once, we use the notation $P^n(\mathbf{x}_0) = P(P^{n-1}(\mathbf{x}_0)) = P \circ P \circ \cdots \circ P(\mathbf{x}_0)$ for the n^{th} *iterate*. Thus, P^n is the composition of P with itself n times. If $\mathbf{x}_n = P(\mathbf{x}_{n-1})$, then $\mathbf{x}_n = P^n(\mathbf{x}_0)$.

p. 215 L -6:

$$\dot{x} = (a + b\, \cos(\theta))\; x - x^3$$

p. 216 L 19:

$$\frac{3\,\dot{x}}{x} - 2a - 2b\, \cos(t)$$

p. 216 L 21-22:

$$\begin{aligned} P'(x_0) &= e^{\int_0^{2\pi} \frac{3\,\dot{x}}{x} - 2a - 2b\, \cos(t)\, dt} \\ &= e^{3\ln|P(x_0)| - 3\ln|x_0|}\, e^{-a4\pi} \end{aligned}$$

p. 219 L -10: Multiplying the third equation by $-(1+x)\ldots$

p. 219 L -7:

$$x^* = \frac{1}{2q}\left((1 - q - 2\,f) + \left[(1 - q - 2\,f)^2 + 4\,q(2\,f + 1)\right]^{1/2}\right).$$

p. 219 L -3:

$$\begin{pmatrix} \epsilon^{-1}(1 - y - 2qx) & \epsilon^{-1}(1 - x) & 0 \\ -y & -(1 + x) & 2f \\ \delta & 0 & -\delta \end{pmatrix}.$$

p. 226 L 7: (The proof of Theorem 6.2.2 needs one more argument.) We have left to show that $\omega(\mathbf{x}_0) = \mathscr{O}(\mathbf{q})$. Assume that $\omega(\mathbf{x}_0) \setminus \mathscr{O}(\mathbf{q}) \neq \emptyset$. By Theorem 5.4.3, the set $\omega(\mathbf{x}_0)$ is connected. Therefore, there would have to exist points $\mathbf{y}_j \in \omega(\mathbf{x}_0) \setminus \mathscr{O}(\mathbf{q})$ that accumulate on a point $\mathbf{y}^*$ in $\mathscr{O}(\mathbf{q})$. Taking a transversal $\mathbf{S}$ through $\mathbf{y}^*$, we can adjust the points $\mathbf{y}_j$ so they lie on $\mathbf{S}$. But, we showed above that $\omega(\mathbf{x}_0) \cap \mathbf{S}$ has to be a single point. This contradiction shows that $\omega(\mathbf{x}_0) \setminus \mathscr{O}(\mathbf{q}) = \emptyset$ and $\omega(\mathbf{x}_0) = \mathscr{O}(\mathbf{q})$, i.e., $\omega(\mathbf{x}_0)$ is a single periodic orbit.

p. 224 L -13: must be strictly *negative*.

p. 229 L 22: $\mu(0) = \mu_0$

p. 230 L 6: (6.9.3b) $\quad \cdots + r^2\, D_4(\theta, \alpha) + O(r^3)$

p. 230 L 9-10:

$$C_{j+1}(\theta,\alpha) = \cos(\theta)B_j^1(\cos(\theta),\sin(\theta),\alpha) + \sin(\theta)B_j^2(\cos(\theta),\sin(\theta),\alpha) \tag{6.9.4}$$

$$D_{j+1}(\theta,\alpha) = -\sin(\theta)B_j^1(\cos(\theta),\sin(\theta),\alpha) + \cos(\theta)B_j^2(\cos(\theta),\sin(\theta),\alpha). \tag{6.9.5}$$

p. 230 L -8: Add a "+" and change 3_2^2 to B_3^2:

$$+\begin{pmatrix} \cos(\theta)\,B_3^1 + \sin(\theta)\,B_3^2 \\ -r^{-1}\sin(\theta)\,B_3^1 + r^{-1}\cos(\theta)B_3^2 \end{pmatrix} + \begin{pmatrix} O(r^4) \\ O(r^3) \end{pmatrix},$$

p. 231 L 4:

$$= \frac{\alpha\, r + r^2\, C_3(\theta,\alpha) + r^3\, C_4(\theta,\alpha) + O(r^4)}{\beta(\alpha) + r\, D_3(\theta,\alpha) + r^2\, D_4(\theta,\alpha) + O(r^3)}$$

p. 236 L9: $J(t) =$, one closing ')' too much.

p. 236 Ex. 6.2.1: Hint: Use a bounding function. (Do not use $\dot r$.)

p. 236 Ex. 6.2.2: A more accurate wording for parts (c) and (d) is as follows:

c. Find the maximum radius r_1 such that all the solutions are crossing outward across a circle of radius r for any $0 < r < r_1$.

d. Find the minimum radius r_2 such that all the solutions are crossing inward across a circle of radius r for any $r > r_2$.

p. 236 Ex. 6.2.3: (This is the same as #2. Change to a new problem.)

Consider the system of differential equations

$$\begin{aligned} \dot x &= 3x + 2y - x(x^2 + y^2) \\ \dot y &= -x + y - y(x^2 + y^2). \end{aligned}$$

a. Classify the fixed point at the origin.
b. Show that $(0, 0)$ is the only fixed point.
c. Calculate $r\dot r$ in terms of x and y.
d. Show that $\dot r$ is positive for small r and negative for large r. Hint: To show the quadratic terms are positive definite (positive for all $(x, y) \neq (0, 0)$), either (i) complete the square or (ii) use the test for a minimum of a function.
e. Prove that the system has a periodic orbit.

p. 238 Ex. 6.2.7c: Use Theorem 6.2.8 to show that, any point $\mathbf{p}_0 = (x_0, y_0)$ in with $x_0 > 0$, $y_0 > 0$, and $\mathbf{p}_0 \neq (5, 12.5)$, $\omega(\mathbf{p}_0)$ must be a periodic orbit. Hints: (i) If $\omega(\mathbf{p}_0)$ contained either $(0, 0)$ or $(30, 0)$ then it must contain both. (ii) Thus, $\omega(\mathbf{p}_0)$ would contain an orbit γ with $\alpha(\gamma) = (30, 0)$ and $\omega(\gamma) = (0, 0)$, i.e., $\gamma \subset W^u(30, 0) \cap W^s(0, 0)$. Since there are no such orbits, this is impossible and $\omega(\mathbf{p}_0)$ must be a periodic orbit.

p. 240 Ex. 6.6.2: A better problem is obtained by using $\dot y = y(-3a + x)$.

p. 241 Ex. 6.7.2(c) : …Show that $P(x) < x$ for $x \geq 2$. …

p. 242 Ex. 6.7.4(b): Show the divergence is $-2y^2$ along the periodic orbit.

p. 247 L. 11-12:

$$\begin{aligned} (x, y, z) &= (\pm\sqrt{b/\sigma}(r + \sigma)/2, 0, (r + \sigma)/2), \\ &= (0, \pm\sqrt{b}(r + \sigma)/2, (r + \sigma)/2), \end{aligned}$$

p. 247 L. 16-17:

$$\frac{(r+\sigma)^2}{8}\left(\frac{b}{\sigma} + 1\right), \quad \frac{(r+\sigma)^2}{8}\,(b+1)\,, \quad \frac{(r+\sigma)^2}{2}, \quad \text{and} \quad 0.$$

For $\sigma > 1$ and $b < 3$, the largest of these is $\dfrac{(r+\sigma)^2}{2}$.

p. 250 L. 12-14: The definition of a Milnor attractor should read as follows: A closed invariant set $\mathbf{A}$ is called a *Milnor attractor* for a flow ϕ provided that (i) the basin of attraction of $\mathbf{A}$, $\mathscr{B}(\mathbf{A})$, has positive measure, and (ii) there is no smaller closed invariant set $\mathbf{A}' \subset \mathbf{A}$ such that the measure of $\mathscr{B}(\mathbf{A}) \setminus \mathscr{B}(\mathbf{A}')$ equals zero.

p. 252 L. 10: It should be “In Examples 5.4.4 and 7.1.9 . . . ”.

p. 257 L. 5: For $r > 1$ and $\sigma > b - 1$,

p. 264 L. 4: $f_2([-1, 0)) = [-1, 1)$

p. 264 L. 14-21: If $x_1 = f_2(x_0)$, $2y_0 = x_0 + 1$, and $2y_1 = x_1 + 1$, then for $0 \le x_0 < 1$,

$$2\,y_1 = x_1 + 1 = f_2(x_0) + 1 = 2\,x_0 - 1 + 1 = 2\,(2\,y_0 - 1), \qquad \text{or}$$
$$y_1 = 2\,y_0 - 1 = 2\,y_0 \pmod 1 = D(y_0).$$

A similar calculation for $x_0 < 0$, also shows that $y_1 = D(y_0)$.

p. 266 L. -11&-10:

$$T(x) = T_2(x) = \begin{cases} 2\,x & \text{for } 0 \le x \le 0.5 \\ 2 - 2\,x & \text{for } 0.5 \le x \le 1. \end{cases}$$

p. 274 L. 10: (Theorem 7.6.4) Then, the $n - 1$ principal Lyapunov exponents . . .

p. 274 L. -10: “direction” should be “direct”

p. 284 L. 3-6: The trapping set $\mathbf{U}$ is closed and bounded, so all the sets $\phi^t(\mathbf{U})$ are closed and bounded for $t \ge 0$. The intersection of closed sets is closed so the attracting set is closed. In fact the sets for $0 < t_1 < t_2$ are nested with $\phi^{t_2}(\mathbf{U}) \subset \phi^{t_1}(\mathbf{U}) \subset \mathbf{U}$. A standard theorem in analysis states that the intersection of nested compact sets is a nonempty compact set, so $\mathbf{A}$ is a nonempty compact set.

p. 287 Exercise 7.1.3: This exercise repeats example 7.1.9. A new replacement problem is given in the list of extra problems.

p. 288 Exercise 7.2.2: This exercise repeats example 7.2.5. A new replacement problem is given in the list of extra problems.

p. 288 Exercise 7.3.2:

$$\begin{aligned} T(x) &= 3x \qquad \pmod 1 \\ &= \begin{cases} 3\,x & \text{for } 0 \le x < \frac{1}{3} \\ 3\,x - 1 & \text{for } \frac{1}{3} \le x < \frac{2}{3} \\ 3\,x - 2 & \text{for } \frac{2}{3} \le x < 1 \\ 0 & \text{for } 1 = y. \end{cases} \end{aligned}$$

Show that this map has sensitive dependence on initial conditions. Hint: If $y_n = x_n + \delta 3^n > x_n$ is on a different side of a discontinuity from x_n, then $y_{n+1} = x_{n+1} + 3(\delta 3^n) - 1$. Try using $r = 1/4$.

p. 297, L. 6: $x_{n+1} = N(x_n) = x_n -$ ... instead of $= x_{n-1}$

p. 308, Theorem 9.1.7.a: “any integer k” should be “any positive integer k”.

p. 308 L. 11: . . . period-n point return . . .

p. 309 L. 10: ... “eventually determine the form” ... –¿ instead of “from”

p. 310 L. 3: ... “maps the subinterval $[1/2, 1]$ across the whole interval $[0, 1]$,” ...

p. 312, L. -10: “The *vertical* line segment from (x_1, x_1) to $(x_1, g(x_1))$ and the *horizontal* line segment”

p. 313 L. -1: All the “f” should be “g”.

p. 314 L. 1: The “f” should be “g”.

p. 314 L. 13: All three “x_1” should be “x_n”.

p. 314, L. -8: period-k point p

p. 315, L. 19: Line 3 of Definition 9.3.1. (line 3): ... “differentiable of order r” ... instead of “or”

p. 315, L. -4: “and all $k \ge 0$” should be deleted.

p. 318, L. -3: $|f^n(x) - p_0| \approx \frac{1}{2}\,|(f^n)''(p_0)| \cdot |x - p_0|^2$.

p. 324, L. 17: $x_+ = \dfrac{1+\sqrt{1-{}^2\!/\!_a}}{2}$

p. 330, L. 6: …if $f''(x) = 0$, then $S_f(x) = N_f'(x)$.

p. 332, L. -17: $f_0'(0) = 1$

p. 334, Theorem 9.5.2(v): It should read

$$\frac{\partial f}{\partial x}(x, m(x)) = 1 + \frac{\partial^2 f}{\partial x^2}(x_0, \mu_0)(x - x_0) + O(|x - x_0|^2).$$

p. 334, L. -5: ... for μ near 1.

p. 335, L. 7: $m(0) = 1$.

p. 337 L. 5: "For $3 < a < 1 + \sqrt{3}$" should be "For $3 < a < 1 + \sqrt{6}$"

p. 338 L. 9: The parameter value for 256 should be $3.569934\ldots$.

p. 340: (Figure 9.5.9) The labels on the horizontal axis should be 3.54, 3.55, 3.56, and 3.57 and not 2.54 etc.

p. 340, L. 13-16: remove the words "odd": Similar windows with other periods occur in the bifurcation diagram, but they are harder to see because they occur over a shorter range of parameters. After the introduction of one of these period-p sinks by a saddle-node bifurcation, there occurs a period doubling cascade of period $p\,2^k$ sinks. The largest windows start with periods 3, 5, and 6.

p. 342, L. -6: Delete "the two maps,".

p. 344, L. -13: $x_n = f^n(x_0)$ instead of $x_n = f(x_0)$

p. 346, L. 9: it should be $= [2\,\sin({}^{\pi s}\!/\!_2)\,\cos({}^{\pi s}\!/\!_2)]^2$.

p. 346, L. 15: $h \circ T(s) = h(2 - 2s)$

p. 347, L. 3: $q_0 = h(p_0)$ not $q_0 = h(q_0)$

p. 347, L. -13: $\left[-\sqrt{(1-a)b^{-1}}, \sqrt{(1-a)b^{-1}}\right]$

p. 354, L. 15: Therefore, we can assume that there exist $a < p < b$ such that $x < f(x) < p$ for $a < x < p$, $p < f(x) < x$ for $p < x < b$, $f(a) = a$ or $f(a) = p$, and $f(b) = b$ or $f(b) = p$.

p. 354 L. 17: (iii) Assume that there exists a $b > p$ such that $f(b) = p$ and $p < f(x) < x$ for $p < x < b$. Let $b > p$ be the smallest such value $b > p$ with $f(b) = p$ and $p < f(x) < x$ for $p < x < b$.

p. 354 L. -16: Case c:

p. 355, Theorem 9.5.2(v): It should read

$$\frac{\partial f}{\partial x}(x, m(x)) = 1 + \frac{\partial^2 f}{\partial x^2}(x_0, \mu_0)(x - x_0) + O(|x - x_0|^2).$$

p. 358 L. 7: The value of $\hat{T}(\theta)$ for ${}^{\pi}\!/\!_2 \le \theta \le \pi$ should be $2(\pi - \theta)$.

p. 359 L. 14: $g^{1+j(x)} \circ h_0 \circ f^{-1-j(x)}(f(x)x)$ should be $g^{1+j(x)} \circ h_0 \circ f^{-1-j(x)}(f(x))$

p. 359, L. -2 : (Exer. 9.1.3) Make a table like 9.1.1 for the number …

p. 360, L. -21 : (Exer. 9.2.1a) In other words, for initial conditions x_0 in different intervals, describe where the iterates $f^n(x_0)$ tend.

p. 365 Exercise 9.4.6(a):

$$a\,x^3 + b\,x^2 + c\,x + d = a\,z^3 + \left(c - \frac{b^2}{3a}\right) z + r = g(z)$$

p. 365, Problem 9.5.5: "and let b_k" should be "and let b_{k+1}".

p. 368, Lemma 10.1.1(b): The added condition should be added to the conclusion that $f((a_1, b_1)) = (a, b)$.

p. 379, L. -8,-7: Definition 10.2.5. A map f from a space $\mathbf{X}$ to itself with an invariant set $\mathbf{A}$ is called *topologically transitive on* $\mathbf{A}$, provided that there is a point x^* such that the orbit $\mathcal{O}_f^+(x^*)$ is dense in $\mathbf{A}$.

p. 382 L. -1:

$$= \bigcap_{j=0}^{n} T^{-j}(\mathbf{I}_{s_j})$$

p. 383, Figure 10.3.2: At the far right, I_{RR} should be I_{RL}.
p. 388 L. 17: (last line of Theorem 10.2.6) .. "by the logistic map G" instead of "tent map G"
p. 397 L. -9: To find a point in $\mathbf{K} \setminus \mathbf{E}$, we take a *ternary* expansion ...
p. 399, L. -16: "and x_1 be the end points of $\mathbf{I}_{s_0 \cdots s_{n-1}}$, which ...
p. 400, Theorem 10.5.10: It should read "The set $\mathbf{S}$ is closed and bounded."
p. 403 L. 7: $0 = a^2 - a - 1$
p. 405 L. 1: The transition graph is reducible, but it does not come from the map given.
p. 405 L. -10: ... based on the transition graph on N symbols, ...
p. 407 L. 2,10: $n \times n$ should be $N \times N$.
p. 407 L. 2,12: "between 1 and n" should be "between 1 and N".
p. 409 L. 23: ... and let $\mathbf{b}$ be a sequence with $b_m = j$
p. 411 L. 8: Let z_i^+ and z_i^- be chosen with $y_{2,i} < z_i^+ < c_i^- < c_i^+ < z_i^- < y_{0,i}$,
p. 414 L. -13: $\mathbf{I}_i = \mathbf{J} \qquad \text{for } q+1 \le i \le n-1$
p. 416 L. -15: Now assume that $\mathbf{s}$ is not an element of the *left*-hand side ...
p. 419 L. -1: closed
p. 420 L. -6: (Exercise 10.1.5b) $f_k(x) = f_n(x)$ for $1/3^n \le x \le 1$ and $f_k(x) \ge f_n(x)$ for $0 \le x \le 1/3^{n+1}$.
p. 428 Theorem 11.1.3: .. $\omega(x^*; T)$ instead of x_0
p. 430 L. 2: A better example is given by

$$W(x) = \begin{cases} \frac{1}{2}\,x & \text{if } x \le 0 \\ 2\,x & \text{if } 0 \le x \le 0.5 \\ 2\,(1-x) & \text{if } 0.5 \le x \le 1 \\ \frac{1}{2}\,(1-x) & \text{if } 1 \le x. \end{cases}$$

p. 430 L. -7: $\mathrm{int}(\mathbf{S}) = \{\mathbf{x} \in \mathbb{R}^n : \text{there is a } r > 0 \text{ such that } \mathbf{B}(\mathbf{x}, r) \subset \mathbf{S}\,\}$
p. 437 L. 24: $c < f^2(b) < b$, so the whole line is

$$a < f(b) < c, \qquad c < f^2(b) < b.$$

p. 438 L. 2: (e) The basin of attraction of $\mathbf{A}$, $\mathscr{B}(\mathbf{A}; f)$, is dense and open in $[a, b]$.
p. 438 L. 6: **h.** If there is no periodic *orbit* ...
p. 438-439: (Example 11.2.19) Consider the example

$$f(x) = \begin{cases} 1.25\,x + 1 & \text{for } x < 0 \\ 1.25\,x - 1.05 & \text{for } x > 0. \end{cases}$$

Notice that $f'(x) = 1.25 < \sqrt{2}$, so Williams theorem does not apply. The ends of the invariant interval are $a = f(0^+) = r_0^+ = -1.05$ and $b = f(0^-) = r_0^- = 1$; they have images in the open interval, $f(a) = -0.3125$, $f(b) = 0.2 \in (a, b)$. A direct check shows that the interval $\mathbf{U} = [-1.1, 1.1]$ is a trapping region for the attracting set $[-1.05, 1]$: $f([-1.1, 1.1]) = [-1.05, 1]$ and $f([-1.05, 1]) = [-1.05, 1]$. The iterates of $r_0^\pm$ are not dense in the whole interval $[-1.05, 1]$, so $[-1.05, 1]$ is not the attractor: See Figure 11.2.5.

To find the attractor $\mathbf{A}$, we need to calculate a few more iterates $f^j(r_0^-) = r_j^-$ and $f^j(r_0^+) = r_j^+$. Because both slopes are positive, any attracting set with 0 in its interior must contain all points just to the right of all the r_j^+ and just to the left of all the r_j^-. Thus, we seek intervals with left end points from the $\{r_j^+\}$ and right end points from the $\{r_j^-\}$. Some iterates are $r_1^- = f(r_0^-) = 0.2$, $r_2^- = f^2(r_0^-) = f(0.2) = -0.8$,

$r_3^- = f^3(r_0^-) = f(-0.8) = 0$, $r_1^+ = f(r_0^+) = -0.3125$, $r_2^+ = f^2(r_0^+) = f(-0.3125) = 0.609375$, and $r_3^+ = f^3(r_0^+) = f(0.609375) = -0.28828125$. The order of these points on the line is as follows:

$$a = r_0^+ < r_2^- < r_1^+ < r_3^+ < r_3^- = 0 < r_1^- < r_2^+ < b = r_0^- = 1.$$

Consider the union of the three closed intervals

$$\mathbf{A} = [r_0^+, r_2^-] \cup [r_1^+, r_1^-] \cup [r_2^+, r_0^-],$$

where $[r_1^+, r_1^-] = [r_1^+, 0] \cup [0, r_1^-]$. Since

$$\begin{aligned} f([r_0^+, r_2^-]) &= [r_1^+, 0], \\ f([r_1^+, 0]) &= [r_2^+, r_0^-], \\ f([0, r_1^-]) &= [r^+, r_2^-] \qquad \text{and} \\ f([r_2^+, r^-]) &= [r_3^+, r_1^-] \subset [r_1^+, r_1^-], \end{aligned}$$

$\mathbf{A}$ is invariant. See Figure 11.2.5. Notice that $r_3^+, r_3^- \in (r_1^+, r_1^-)$ are in the interior of $\mathbf{A}$. Since the end points of $\mathbf{A}$ eventually get mapped into its interior, no periodic orbit is contained entirely on the end points, a trapping region for $\mathbf{A}$ can be found, and $\mathbf{A}$ is an attracting set.

We want to show that $\mathbf{A}$ is indecomposable, and so an attractor. In the general situation of Theorem 11.2.17, an attracting set $\mathbf{A}$ that contains the single discontinuity c is indecomposable provided that the forward orbit of an arbitrarily small open interval $(-\delta + c, \delta + c)$ about c covers all of $\mathbf{A}$. In our example, the one-sided small intervals $(-\delta, 0] \subset [r_1^+, 0]$ have a forward orbits that cover $\mathbf{A}$. First we check the iterates of the whole interval $[r_1^+, 0]$:

$$\begin{aligned} f([r_1^+, 0]) &= [r_2^+, r_0^-], \\ f([r_2^+, r_0^-] &= [r_3^+, r_1^-] \supset [0, r_1^-], \\ f([0, r_1^-]) &= [r_0^+, r_2^-], \qquad \text{and} \\ f([r_0^+, r_2^-]) &= [r_1^+, 0]. \end{aligned}$$

Since $f^4([r_1^+, 0] \supset [r_1^+, 0]$ and $f^4(0^-) = 0$, there is a subinterval $[x_1, 0]$ in $[r_1^+, 0]$ such that $f^4([x_1, 0]) = [r_1^+, 0]$ and f^4 is an expansion from $[x_1, 0]$ onto $[r_1^+, 0]$. Because $f^4\big|_{[x_1,0]}$ is an expansion, for any smaller interval $(-\delta, 0] \subset [0, x_1]$, there is an an integer j such that $f^{4j}((-\delta, 0]) \supset [x_1, 0]$ and $f^{4j+4}((-\delta, 0]) \supset [r_1^+, 0]$. Thus, the forward orbit of any small $(-\delta, 0]$ must contain the entire interval $[r_1^+, 0]$ and its iterates, hence all of $\mathbf{A}$. It follows that $\mathbf{A}$ is generated by iterates of an arbitrarily small interval $(-\delta, 0]$ and must be indecomposable and a chaotic attractor made up of three intervals.

In the gaps (r_2^-, r_2^+) and (r_1^-, r_2^+) of points left out of the attractor, …

p. 439 L. -11: $= \frac{9}{16}\, x + \frac{1}{5}$

p. 444 L. 3: (i) If there is no periodic *orbit* …

p. 445 L. -1: $|\ln(|f'(x_j)|) - \ln(|f'(p_j)|)| < \epsilon \qquad \text{for all } \ j \geq N.$

p. 448 L. -7: $s_n \ldots s_{n+N-1} = L^N$

p. 449 L. 1: Therefore, $(\pi/2)\sin(\pi y_n) \geq (\pi/2)\,\pi 2^{1-N}/2 = \pi^2 2^{-(N+1)} \ldots$

p. 449 L. 3: $\geq \lim_{N\to\infty} \dfrac{\ln\left(\pi^2\, 2^{-N-1}\right)}{2^{N-1}}$

p. 449 L. -5: … with $\omega(x_0; f) = \mathbf{A}$ and $\ell(x_0, f) > 0$.

p. 451 L. 1: The *support* $\text{supp}(\mu)$ of a probability measure μ is defined to be

p. 453 L. -10: (First printing only) For a closed set $\mathbf{S}$,

p. 455 L. -6: First if $\mathbf{U}_1$ and $\mathbf{U}_2$ are disjoint *open sets*, then …

p. 459 L. -5: $\mathbf{F}\begin{pmatrix} r_0 \\ \theta_0 \end{pmatrix} =$

p. 472, Problem 11.2.7: (Alternative to 11.2.4) Consider the map

$$f(x) = \begin{cases} -1.2\,x - 0.288 & \text{for } x < 0 \\ -1.2\,x + 0.2 & \text{for } x \geq 0. \end{cases}$$

a. Calculate the first four iterates of 0^- and 0^+ ($r^-, r_1^-, r_2^-, r_3^-, r^+, r_1^+, r_2^+, r_3^+$) and give their order them on the line.
b. Show that there is a set A made up of three intervals, and that A is invariant. The middle interval contains 0 in its interior. *Hint:* Because both sides of f have negative slope, any attracting set with 0 in its interior contains all points just to the right of $\{\, r^-, r_1^+, r_2^-, r_3^+ \,\}$ and all points just to the left of $\{r^+, r_1^-, r_2^+, r_3^- \,\}$.
c. Show that the forward iterates of the end points of A eventually fall into the interior of A. Conclude that A has a trapping region and is an attracting set.
d. Show that there is an interval $[0, \delta]$ that has an iterate that covers itself by an expansion and takes 0 to 0. Argue why the expanding set A of part (b) is indecomposable and a chaotic attractor.
e. Find the two fixed points, one positive and one negative. Show that the fixed points are not in the set A found in the last part, but are in the gaps between the intervals.

p. 473, Problem 11.3.1: $p_a = {}^{13}\!/_{19}$, not ${}^{16}\!/_{19}$.

p. 474, Problem 11.4.5, L -8: $2\,x - \frac{5}{4}$ for $\frac{3}{4} < x \leq 1$.

p. 476: Line 1 after Example 12.1.1: one "with"

p. 476 L. -1: A *linear sink* is allowed to have complex as well as real eigenvalues as long as every eigenvalue of $\mathbf{A}$ has $|\lambda| < 1$.

p. 492 L. -16: It has characteristic equation $\lambda^2 + \lambda - a\,\cos(x_a) = 0 \ldots$

p. 496 L. 26: (First printing only) Should be

$$D(\mathbf{F}^{-1})_{(\mathbf{y})} = \left(D\mathbf{F}_{(\mathbf{x})}\right)^{-1},$$

p. 510 L. 22-3: (In the definition of irreducible) $j_q = i_2$ not j_m. And "the (i_1, i_2)-entry of $\mathbf{M}^q$ is nonzero."

p. 511 L. 14: Thus, $2v_1 = v_3$, $2v_2 = -3v_3, \ldots$

p. 513 L. -9: Case (ii)

p. 519 L. 1: At the n^{th} stage, S_n is the size of the susceptible population and I_n is the size of the infected population.

p. 526 L. 11: (First printing only) (12.5.2) The iterate should be

$$I_{n+1} = I_n \left(1 - \gamma + \frac{\alpha}{N}\, S_n\right)$$

p. 526 L. 16: (First printing only) (12.5.2(b)) $p = 1 - \gamma + \alpha$

p. 534, Figure 13.1.4(b): $\mathbf{B}_{.10}$ should be above $\mathbf{B}_{.11}$, i.e., interchange the two labels in the figure.

p. 546: (Definition 13.2.8) Several "∂" should be replaced by "bd". Also

$$\partial^{\text{in}}(\mathbf{R}) = \text{bd}\left(\bar{\mathbf{B}}^{n_1}(\mathbf{0}, r_1)\right) \times \bar{\mathbf{B}}^{n_2}(\mathbf{0}, r_2),$$
$$\partial^{\text{out}}(\mathbf{R}) = \bar{\mathbf{B}}^{n_1}(\mathbf{0}, r_1) \times \text{bd}\left(\bar{\mathbf{B}}^{n_2}(\mathbf{0}, r_2)\right).$$

p. 548 L. 19: $\mathbf{F}\left(\partial^{\text{out}}(\mathbf{R}_i)\right) \cap \text{int}\left(\mathbf{R}_j\right) = \emptyset$.

p. 548 L. 21: $\phi_i(\{\, \mathbf{x} \times \bar{\mathbf{B}}^{n_2}(\mathbf{0}, 1))\,\})$

p. 558: (Figure 13.3.1) The following figure shows more of the stable and unstable manifold for a saddle with a transverse homoclinic point.

p. 574 L. 4: "$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$," should be "$\ell_1 \geq \ell_2 \geq \cdots \geq \ell_n$,"

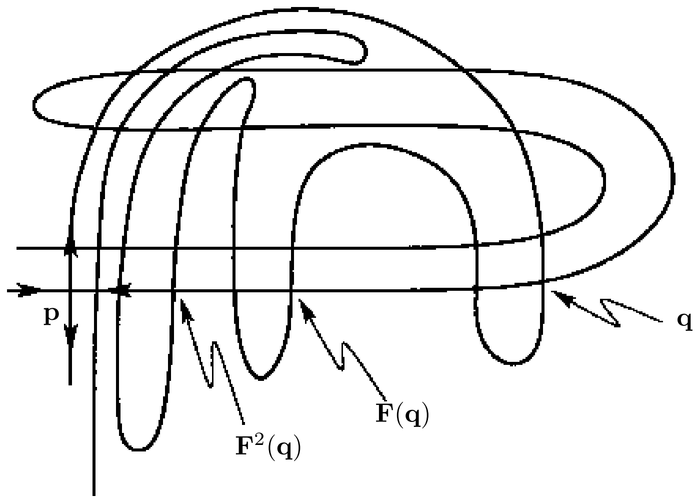

FIGURE 1. A transverse homoclinic point $\mathbf{q}$

p. 577: (Example 13.6.4) To obtain the map $f(r)$ as given in polar coordinates, we need

$$\mathbf{F}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos(2\pi\,\omega) & -\sin(2\pi\,\omega) \\ \sin(2\pi\,\omega) & \cos(2\pi\,\omega) \end{pmatrix} \begin{pmatrix} 3\,x\,(1+2\sqrt{x^2+y^2})^{-1} \\ 3\,y\,(1+2\sqrt{x^2+y^2})^{-1} \end{pmatrix}$$

Then the second coordinate function in polar coordinates should be $\theta + 2\pi\,\omega \pmod{2\pi}$.

p. 584: (First printing only) (13.2.1(e)) The second map labeled "e" should be part "f".

p. 587 Ex 13.4.3: $\mathbf{R}_2 = \{\,(t,z) \in \mathbf{N} : {}^2\!/_3 \le t \le 1\,\}$ instead of $\mathbf{R}_0$.

p. 591, L. -4: (The count is slightly off because the intervals are half open.)
of the intervals. This leaves k points $1,\ldots,k$ that are ...

$$N(r_k,\mathbf{S}) = k + 1 + k = 2k + 1.$$

p. 591 L. 1,: It would be better to define $N'(r,\mathbf{S})$ using *closed* boxes. Similarly, $N''(r,\mathbf{S})$ is defined using *closed* balls.

p. 593, Theorem 14.1.7: A right parenthesis is missing in the third display. It is probably better to state this theorem with $N'(r_k,\mathbf{S})$ rather than $N(r_k,\mathbf{S})$.

p. 597 L. 8: For small $r > 0$,

p. 605 L. 9: $\dfrac{\ln(3)}{\ln(2)} > 1.$

p. 607: Definition 14.3.9: The δ should all be d.

p. 609: (After Theorem 14.3.15) **Remark**: The unique fixed set $\mathbf{A}$ in Theorem 14.3.15 is the same set as the attractor $\mathbf{A}$ in Theorem 14.3.7. The construction in Theorem 14.3.7 is closer to the treatment of attractors from trapping regions in the earlier part of the book and is less abstract. The contraction in terms of the Hausdorff metric considered in Section 14.3.1 is more mathematically elegant and is presented in many other books.

p. 614 L. 11: The last term in the display should be

$$\le \frac{\ln({}^1\!/_{r_{j+1}})}{\ln({}^1\!/_{r_j})}\,\frac{N'(r_{j+1},\mathbf{S})}{\ln({}^1\!/_{r_{j+1}})}.$$

Note that $\ln({}^1\!/_{r_j})$ replaces $\ln({}^1\!/_j)$.

p. 623: (First printing only) The label for Figure 14.4.1 should read "Fractals for Exercise 14.3.7."

p. 634 L -12:

$$p(x) = (x - \lambda_1)^{m_1} \cdots (x - \lambda_q)^{m_q},$$

p. 635 L 11: $\mathbf{A}\mathbf{v}^{(2)} = \lambda_k \mathbf{v}^{(2)} + \mathbf{v}^{(1)}$, and

Thanks goes to the following people who have pointed out many errors or typographical mistakes: John Alongi, Keith Burns, William Floyd, Héctor Lomelí, and Konrad Schrempf.