Math C13-1	Final	12/10/92	C. Robinson

Closed book. You may use hand calculators.

1. (40 Points) Let $f(x) = 2x - 4x^3$.
   (a) Find the fixed points and classify them as attracting, repelling, or neither.
   (b) Use the stair step method to determine the behavior of iterates $f^n(x)$ for all $x \in (-\infty, \infty)$. Give the basin of attraction for each of the fixed points.

2. (20 Points) For $\mu = 3.839$, the quadratic map $Q_\mu(x) = \mu x(1-x)$ has a period three cycle at approximately the points $x_0 = 0.959$, $x_1 = Q_\mu(x_0) = 0.150$, and $x_2 = Q_\mu^2(x_0) = 0.489$. Asssuming these numerical values are exactly the points on the period three cycle, determine the stability of this cycle.

3. (80 Points) Consider the quadratic map $Q_5(x) = 5x(1-x)$.
   (a) Prove that Q_5 has sensitive dependence on initial conditions on the whole real line (not just the Cantor set C_5).
   (b) Prove that Q_5 is transitive on its invariant Cantor set C_5. (If you use facts about another map, verify these facts.)
   (c) How many points of period 7 does $Q_5(x)$ have?

4. (30 Points) Let $f(x) = x^3$ and $g(y) = \frac{1}{4}y^3 + \frac{3}{2}y^2 + 3y$. Find an affine map $y = h(x) = ax + b$ which conjugates f and g. Verify that your map h works. (Note that f has fixed points at -1, 0, and 1, and g has fixed points at -4, -2, and 0.)

5. (30 Points) Let $f(x) = 3x + \sin(x)$.
   (a) Find the Lyapunov exponent for $x = 0$.
   (b) Show that for any real x, the Lyapunov exponent $\lambda(x)$ satisfies $ln(2) \leq \lambda(x) \leq ln(4)$.