⑥ (a) The set $|z| \le 1$ is taken in the second coordinate to

$$\left| \tfrac{1}{4} z + \tfrac{1}{2} e^{2\pi t i} \right| \le \tfrac{1}{4}|z| + \tfrac{1}{2} \le \tfrac{3}{4} < 1$$

So the set $N$ is mapped inside itself.

(b) $\Delta = \bigcap_{k=0}^{\infty} F^k(N)$ is an attracting set

because it is the intersection of iterates of a trapping region.

It can be shown there are no smaller attracting sets, so it is an attractor.

The map expands in the first coordinate by 2. So, just like the doubling map, $F$ has sensitive dependence on initial conditions when restricted to $\Delta$.

Thus chaotic attractor by definition in class.

The Lyapunov exponents are $\ln 2 > 0$, $\ln(\frac{1}{4}) < 0$, and $\ln(\frac{1}{4}) < 0$.

Thus, orbits in $\Delta$ which are not asymptotically periodic are chaotic orbits by definition in ASY.

It can be shown there is a point $p$ with $\omega(p) = \Delta$, so $\Delta$ is a chaotic attractor by definition in ASY.