Math C13 Final Exam: 9 am, Tuesday, March 17, 1998.

Name:____

You have 2 hours to answer the following 6 questions. Point-values are marked, for a total of 200. Write all work in the space provided. No calculators or notes. Have fun!

1. (30 points) Let $h_{a,b}(x, y) = (a - x^2 + by, x)$. In the space below, draw the image of the square $S = \{(x, y)| - 1 \leq x \leq 1, -1 \leq y \leq 1\}$ under $h_{a,b}$ for the parameter values specified.

   file=square.ps,height=2.0 in,width=2.0in
   $a = 1, \qquad b = 0$

   file=square.ps,height=2.0in,width=2.0in
   $a = 0, \qquad b = 1/2$

2. (35 points) The graph of f^2 is shown below. Explain why f must have at least three fixed points. Identify them on the graph.

   file=final3132-98-fig.ps,height=2.4 in,width=2.8 in

3. (30 points) Suppose that a continuous function f is defined on the interval $[1, 7]$, passes through the points $(1, 4), (2, 7), (3, 6), (4, 5), (5, 3), (6, 2)$, and $(7, 1)$, and is linear in between. For which n does f have a periodic point of period n?

4. (40 points) Let $a > 0$ and define $f_a : \mathbf{R}^2 \to \mathbf{R}^2$ by the formula

$$f_a(x, y) = (1 - ax^2 + y, x).$$

   (a) Find all period-two points for f_a.

   (b) Find all values of a for which the the period-two cycle is of saddle type.

5. (35 points) Let $F : \mathbf{R}^2 \to \mathbf{R}^2$ be given by

$$F(x, y) = (\frac{1}{2}x, x + \frac{1}{2}y).$$

(a) Show that all of the eigenvalues of F are less than 1 in modulus.

(b) Find a vector w such that $\|F(w)\| > \|w\|$.

(c) Find a real number $c > 0$ such that $\|F(v)\|^2 \leq c\|v\|^2$, for all $v \in \mathbf{R}^2$.

6. (30 points) Let $f_a(x) = a\sin(x)$, for $0 \leq x \leq 2\pi$, where $0 < a < 2\pi$. Determine the maximal number of attracting periodic orbits of f_a.