(1) (25 Points) The doubling map is defined by

$$D(x) = 2x \pmod 1.$$

(a) Determine the complete orbit for each of the points 1/3, 1/5, and 1/10. Indicate whether each of these points is periodic, eventually periodic, or neither.
(b) Determine how many points the map D has of the least period 1, 2, 3, and 6.

(2) (25 Points) Let $f(x) = \dfrac{x^3}{2} + \dfrac{x}{2}$.
(a) Find the fixed points and classify them as attracting, repelling, or neither.
(b) Use the cobweb plot analysis to determine the dynamical behavior of all points in $\mathbb{R}$. Describe the orbits using words as well as by the plot.

(3) (25 Points) Let $G(x) = 4x(1 - x)$. Find the map $f(y)$ that is conjugate to G via the map $y = C(x) = 2x - 1$. Note that $x = C^{-1}(y) = \dfrac{y+1}{2}$.

(4) (25 Points) Consider the map $F(x) = rx(1 - x)$ for $r = 3.2$. The fixed points are $\mathbf{0}$ and $\mathbf{p} = 11/16$, both of which are repelling. It has a stable period-2 orbit, $\{\mathbf{q}_1, \mathbf{q}_2\}$ where

$$\mathbf{q}_1 = \frac{1 + r - (r^2 - 2r - 3)^{\frac{1}{2}}}{2r} \approx 0.5130$$

$$\mathbf{q}_2 = \frac{1 + r + (r^2 - 2r - 3)^{\frac{1}{2}}}{2r} \approx 0.7995.$$

All other points $x_0 \in (0, 1)$ are either (i) eventually periodic with period 1 (eventually landing on the fixed point $\mathbf{p}$), or (ii) asymptotic to the period-2 orbit $\{\mathbf{q}_1, \mathbf{q}_2\}$. (You do not need to show any of these facts.)
(a) Determine the Lyapunov exponents $h(x_0)$ for all the points in $[0, 1]$. Explain why your answer is correct.
(b) Are there any chaotic orbits for points in $[0, 1]$? Explain your answer.