Math 313-1 Test #1 1/28/03

①

n	# $Fix(f^n)$	Lower Period	# Per(n)	# orbits
1	3	0	3	3
2	9	3	6	3
3	27	3	24	8
4	81	3+6 = 9	72	18
5	243	3	240	48
6	729	3+6+24 = 33	696	116

ⓑ $f(\frac{1}{6}) = \frac{1}{2}$ $f(\frac{1}{2}) = \frac{1}{2}$. $\frac{1}{6}$ is eventually fixed.

$f(\frac{1}{4}) = \frac{3}{4}$ $f(\frac{3}{4}) = \frac{1}{4}$ period-2

② ⓐ $f(x) - x = x^3 + \frac{5}{9}x - x = x^3 - \frac{4}{9}x = 0.$ $x = 0, \pm \frac{2}{3}$

$= x(x^2 - \frac{4}{9}) = 0.$

$f'(x) = 3x^2 + \frac{5}{9}$ $f'(0) = \frac{5}{9}$ 0 is ~~stable~~ attracting

$f'(\pm\frac{2}{3}) = 3 \cdot \frac{4}{9} + \frac{5}{9} = \frac{17}{9} > 1$ $\pm\frac{2}{3}$ are each repelling

ⓑ the graph $f(x) > x$ $-\frac{2}{3} < x < 0$

$< x$ $0 < x < \frac{2}{3}$.

By theorem given in book, the basin of 0 contains $(-\frac{2}{3}, \frac{2}{3})$.

Outside this interval, points go to $\pm\infty$.

$\therefore \mathcal{B}(0) = (-\frac{2}{3}, \frac{2}{3})$.