313-1 Test #1 1/28/03

(3) $f'(x) = 3x^2 - \frac{5}{4}$.

$f'(\pm\frac{1}{2}) = 3 \cdot \frac{1}{4} - \frac{5}{4} = -\frac{1}{2}$.

$\therefore \left|(f^2)'(\pm\frac{1}{2})\right| = \left|f'(\frac{1}{2})f'(-\frac{1}{2})\right| = \left|(-\frac{1}{2})^2\right| = \frac{1}{4} < 1.$

Therefore the orbit $\{-\frac{1}{2}, \frac{1}{2}\}$ is attracting.

(4) (a) $f(x) = x^3 - \frac{9}{16}x$

$f'(x) = 3x^2 - \frac{9}{16}$

$f''(x) = 6x$

$f'''(x) = 6$.

$$S_f(x) = \frac{6\left(3x^2 - \frac{9}{16}\right) - \frac{3}{2}(6x)^2}{\left(3x^2 - \frac{9}{16}\right)^2} = \frac{-\frac{27}{8} + 18x^2 - 54x^2}{\left(3x^2 - \frac{9}{16}\right)^2}$$

$$= \frac{-\frac{27}{8} - 36x^2}{\left(3x^2 - \frac{9}{16}\right)^2} < 0$$

(b) $0 = f'(x) = 3x^2 - \frac{9}{16}$ $\quad x^2 = \frac{3}{16}$ $\quad x = \pm\frac{\sqrt{3}}{4}$ critical points.

(c) $S_f(x) < 0$ and $B(0) \subset (-\frac{5}{4}, \frac{5}{4})$.

Therefore the basin does not extend to either $\pm\infty$. Therefore it must contain at least one critical point.

Since $f(-x) = -f(x)$, if either critical point goes to 0 then the other does as well.

Therefore both $\pm\frac{\sqrt{3}}{4}$ are in $B(0)$.