

(d) Fix a point \mathbf{p} . For $\mathbf{q} \in W^u(\mathbf{p}) \setminus \{\mathbf{p}\}$, the distance $d(f_A^k(\mathbf{p}), f_A^k(\mathbf{q})) \geq \lambda^k d(\mathbf{p}, \mathbf{q})$ as long as the distance stays less than one half. The quantity $\lambda^k d(\mathbf{p}, \mathbf{q})$ grows as $k > 0$ increases, so this distance gets bigger than $1/(2\lambda)$ for some $k > 0$. This proves that f_A has sensitive dependence. Similarly, for $\mathbf{q} \in W^s(\mathbf{p}) \setminus \{\mathbf{p}\}$, the distance $d(f_A^k(\mathbf{p}), f_A^k(\mathbf{q})) \geq \mu^k d(\mathbf{p}, \mathbf{q})$ grows as $k < 0$ becomes more negative, so it becomes bigger than $\mu/2$ for some $k < 0$.

Finally, there is an $\delta_0 > 0$ such that for any point $\mathbf{q} \neq \mathbf{p}$ with $d(\mathbf{p}, \mathbf{q}) < \delta_0$, the two points $\mathbf{q}_u \in W^u(\mathbf{p}) \cap W^s(\mathbf{q})$ and $\mathbf{q}_s \in W^s(\mathbf{p}) \cap W^u(\mathbf{q})$ are within $1/(2\lambda^2)$ and $\mu^2/2$ respectively of \mathbf{p} . As least one of these two points is distinct from \mathbf{p} ; let us take the case when $d(\mathbf{p}, \mathbf{q}_u) \geq d(\mathbf{p}, \mathbf{q}_s)$. Because the stable and unstable manifolds are parallel lines in the universal covering space, $d(\mathbf{p}, \mathbf{q}_u) = d(\mathbf{q}_s, \mathbf{q})$ and $d(\mathbf{p}, \mathbf{q}_s) = d(\mathbf{q}_u, \mathbf{q})$. There is a first $k_1 \geq 1$ such that $d(f^{k_1}(\mathbf{p}), f^{k_1}(\mathbf{q}_u)) \geq 1/(2\lambda)$. Then

$$\begin{aligned} d(f^{k_1}(\mathbf{p}), f^{k_1}(\mathbf{q})) &\geq d(f^{k_1}(\mathbf{p}), f^{k_1}(\mathbf{q}_u)) - d(f^{k_1}(\mathbf{q}_u), f^{k_1}(\mathbf{q})) \\ &\geq \frac{1}{2\lambda} - \mu d(\mathbf{q}_u, \mathbf{q}) \\ &\geq \frac{1}{2\lambda} - \mu d(\mathbf{p}, \mathbf{q}_s) \\ &\geq \frac{1}{2\lambda} - \mu \left(\frac{1}{2\lambda^2} \right) \\ &= \left(1 - \frac{\mu}{\lambda} \right) \left(\frac{1}{2\lambda} \right). \end{aligned}$$

Similarly, if $d(\mathbf{p}, \mathbf{q}_s) \geq d(\mathbf{p}, \mathbf{q}_u)$, then for some $k_1 < 0$

$$d(f^{k_1}(\mathbf{p}), f^{k_1}(\mathbf{q})) \geq \left(1 - \frac{\mu}{\lambda} \right) \left(\frac{\mu}{2} \right).$$

Therefore, f_A is expansive with expansive constant

$$\min \left\{ \delta_0, \left(1 - \frac{\mu}{\lambda} \right) \frac{1}{2\lambda}, \left(1 - \frac{\mu}{\lambda} \right) \frac{\mu}{2} \right\}$$

This proves part (d).