ERRATA AND ADDITIONS FOR THE 2008 PRINTING OF THE SECOND EDITION OF DYNAMICAL SYSTEMS: STABILITY, SYMBOLIC DYNAMICS, AND CHAOS

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p. 33: (Section 2.4.3) Explanation: S. Zeller and M. Thaler ("Almost sure excape from the unit interval under the logistic map", *Amer. Math. Monthly* **108** (2001), pages 155 − 158.) have a simpler proof based on the earlier thesis of S. Zeller ("Chaosbegriffe der topologishen Dynamik", Dipolmarbeit, Salzburg, 1991). The map

$$y = \phi(x) = \frac{2}{\pi} \arcsin \sqrt{x}$$

is a conjugacy between $F_4(x)$ and $g_4(y) = 1 - |1 - 2y|$, $g_4(y) = \phi \circ F_4 \circ \phi^{-1}(y)$. For $\mu > 4$, $F_\mu([0, 1]) = [0, \frac{\mu}{4}]$, so it is natural to scale ϕ by the factor $\frac{\mu}{4}$ to investigate F_μ. Let $\phi_\mu(x) = \frac{\mu}{4}\,\phi(\frac{4}{\mu}x)$. Define the map g_μ by

$$g_\mu(y) = \phi_\mu \circ F_\mu \circ \phi_\mu^{-1}(y).$$

Then a simple calculation shows that $|g'_\mu(y)| \geq \sqrt{\mu}$ for all $y \in [0, \phi_\mu(a)]$, so F_μ has can invariant Cantor set. The proof is a follows. Let $y = \phi_\mu(x)$ for $x \in [0, 1)$. Because ϕ is a conjugacy of F_4 and g_4,

$$\phi'(F_4(x))\, F'_4(x) = (2\,\mathrm{sign}(1 - 2x))\, \phi'(x).$$

Also, $F'_\mu(x) = \frac{a}{4} F'_4(x)$. Thus

$$\begin{aligned} g'_4(y) &= \frac{\phi'_\mu(F_4(x))\, F'_\mu(x)}{\phi'_\mu(x)} = \frac{a\, \phi'(F_4(x))\, F'_4(x)}{4\, \phi'_\mu(x)} \\ &= \mathrm{sign}(1 - 2x))\, \frac{\mu\, \phi'(x)}{2\, \phi'_\mu(x)} \\ &= \mathrm{sign}(1 - 2x))\, \sqrt{\mu}\, \Big(\frac{1 - \frac{4}{\mu}}{1 - x}\Big)^{\frac{1}{2}}. \end{aligned}$$

Since $\frac{4}{\mu} < 1$, the last term is greater than $\sqrt{\mu}$.

p. 38: (Section 2.5) In this section, we show that the dynamics of F_μ on Λ can be understood in terms of a map on a symbol space made up by points which are sequences of 1's and 2's. The map on the symbol space is said to give the *symbolic dynamics* for the map. At least in a theoretical way, we can determine the periodic points. We also want to show that there are points whose orbit is dense in the cantor set Λ and points with other complicated dynamics. By introducing symbols to describe the location of a point, the dynamics of a point in the Cantor set can be determined by means of a sequence of these symbols. Because many different patterns of symbols can be written down, points with many different types of dynamics can be shown to exist.

p. 50: (L. -23) Explanation: The covering space $\mathbb{R}$ of S^1 can be thought of as measuring the angle without reducing modulo 2π, or modulo 1, in the coordinates on $\mathbb{R}$. Thus, the points t, $t+1$, and $t+2$ in $\mathbb{R}$ all represent the same point in $\S^1$. In the same way, the lift of $f : S^1 \to S^1$ to $F : \mathbb{R} \to \mathbb{R}$ gives the new location without reducing modulo 1. The difference $F(t) - t$ is the amount the point is move around the circle without reducing modulo 1.

p. 50: (L. -9) Explanation: Let $F_\lambda(t) = t + \lambda$ be the rigid rotation. Then the change of angle, $F_\lambda(t)(t) - t = \lambda$ is the same for any point. For an arbitrary homeomorphism of S^1, the change $F(t) - t$ can vary with the point t. The quantity

$$F^n(t) - t = [F^n(t) - F^{n-1}(t)] + [F^{n-1}(t) - F^{n-2}(t)] + \cdots + [F(t) - t]$$

is the total change of angle by the n^{th}-iterate without reducing modulo 1. The average change of angle for one iterate by the first n-iterates is

$$\frac{1}{n}\{F^n(t) - t\} = \frac{1}{n}\{[F^n(t) - F^{n-1}(t)] + [F^{n-1}(t) - F^{n-2}(t)] + \cdots + [F(t) - t]\}.$$

Taking the limit as n goes to infinity, $\lim_{n\to\infty} \frac{1}{n}\{F^n(t) - t\}$ gives the average change of angle for one iterate along the whole orbit. This last limit is used to define the rotation number of the map on the circle.

p. 79: (L. -8) there is an allowable word $\mathbf{w}$ such that

p. 96: (L 5) Explanation: The norm of a matrix can be calculated in terms of an eigenvalue of a related matrix. Notice that

$$|A\mathbf{x}|^2 = (A\mathbf{x})^t A\mathbf{x} = \mathbf{x}^t A^t A\mathbf{x}.$$

The maximum of this quantity as $\mathbf{x}$ varies over unit vectors is the square of the norm of A. The matrix A^tA is symmetric and so has real eigenvalues. If λ_1 is the largest eigenvalue with unit eigenvector $\mathbf{v}^1$ then

$$\mathbf{v}_1^t A^t A \mathbf{v}_1 = \mathbf{v}_1^t \lambda_1 \mathbf{v}^1 = \lambda_1.$$

Therefore the norm of A is the square root of the largest eigenvalue of A^tA, $\|A\| = \sqrt{\lambda_1}$.

p. 114: (L. 6) $\mathbf{v} \in V^u$ should be $\mathbf{v} \in V^c$: "as $t \to \pm\infty$, so $\mathbf{v} \in V^c$."

p. 134: (Line -7 to -4) Replace with: "If U is a region where $f(\mathbf{x})$ is defined and C^1 and $V \subset U$ is a compact subset, then we can let $K = \sup\{\|Df_{\mathbf{x}}\| : \mathbf{x} \in V\}$. By the Mean Value Theorem,

$$|f(\mathbf{x} - f(\mathbf{y})| \le K|\mathbf{x} - \mathbf{y}|$$

if the line segment from $\mathbf{x}$ to $\mathbf{y}$ is contained in V."

p. 143: (Line 7–9) For $\mathbf{x}_0 \in U$ take $b > 0$ such that the closed ball $\bar{B}(\mathbf{x}_0, b) \equiv \{\mathbf{x} : |\mathbf{x} - \mathbf{x}_0| \le b\} \subset U$. The function f is Lipschitz ... for all $\mathbf{x}, \mathbf{y} \in \bar{B}(\mathbf{x}_0, b)$.

p. 389: (Line 6) The way to calculate the limits of the wedge product is to start with an orthonormal basis $\{\mathbf{v}^{0,1}, \ldots, \mathbf{v}^{0,m}\}$ of tangent vectors at $\mathbf{x}^0 = \mathbf{x}$. Let $\mathbf{x}^k = f^k(\mathbf{x})$. Assume by induction that we have defined an orthonormal basis $\{\mathbf{v}^{k-1,1}, \ldots, \mathbf{v}^{k-1,m}\}$ at $\mathbf{x}^{k-1}$. Applying the derivative at x^{k-1}, let $\mathbf{w}^{k,j} = Df_{\mathbf{x}^{k-1}}\mathbf{v}^{k-1,j}$ be the image vectors. Apply the Gram-Schmidt process to construct a basis of perpendicular vectors:

$$\begin{aligned}
\mathbf{z}^{k,m} &= \mathbf{w}^{k,m} \\
\mathbf{z}^{k,m-1} &= \mathbf{w}^{k,m-1} - \frac{\mathbf{w}^{k,m-1} \cdot \mathbf{z}^{k,m}}{|\mathbf{z}^{k,m}|^2}\mathbf{z}^{k,m} \\
\mathbf{z}^{k,j} &= \mathbf{w}^{k,j} - \sum_{i=j+1}^{m} \frac{\mathbf{w}^{k,j} \cdot \mathbf{z}^{k,i}}{|\mathbf{z}^{k,i}|^2}\mathbf{z}^{k,i} \qquad \text{for } 1 \le j \le m-1.
\end{aligned}$$

We get an orthonormal basis of vectors at x^k by letting

$$\mathbf{v}^{k,j} = \frac{\mathbf{z}^{k,j}}{|\mathbf{z}^{k,j}|}.$$

This completes the induction process. The multiplicative factor of the j^{th}-vector is

$$r_j^{(k)} = |\mathbf{w}^{1,j}| \cdots |\mathbf{w}^{k,j}|.$$

The volume of the parallelograms spanned by $\{\mathbf{z}^{k,m-j+1}, \ \dots, \ \mathbf{z}^{k,m}\}$ is the same as that spanned by the $\{\mathbf{w}^{k,m-j+1}, \ \dots, \ \mathbf{w}^{k,m}\}$, which is $r^{(k)}_{m-j+1} \cdots r^{(k)}_m$. Thus the growth rate of this volume as k goes to infinity is

$$\begin{aligned}\lambda_{m-j+1} + \cdots \lambda_m &= \lim_{k\to\infty} \frac{1}{k} \log(r^{(k)}_{m-j+1} \cdots r^{(k)}_m) \\ &= \lim_{k\to\infty} \frac{1}{k} \log(r^{(k)}_{m-j+1}) + \cdots \lim_{k\to\infty} \frac{1}{k} \log(r^{(k)}_m),\end{aligned}$$

and

$$\begin{aligned}\lambda_{m-j+1} &= \lim_{k\to\infty} \frac{1}{k} \log(r^{(k)}_{m-j+1}) \\ &= \lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} \log(|\mathbf{w}^{i,m-j+1}|).\end{aligned}$$

p. 421: In the proof of Theorem 5.4, if we assume that $\mathbb{R}(f)$ is hyperbolic, then it is possible to take the chain componenets rather than the sets $\mathrm{cl}(H_{\mathbf{p}})$ in the decomposition.

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