A UNIFORM BOUND ON COMMON PREPERIODIC POINTS FOR QUADRATIC POLYNOMIALS

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Abstract. In this article, we prove the existence of a uniform bound on the number of preperiodic points that can be shared by two quadratic polynomials \( f_1(z) = z^2 + c_1 \) and \( f_2(z) = z^2 + c_2 \), over all pairs \( c_1 \neq c_2 \) in \( \mathbb{C} \). We reduce the problem to the case of \( c_1 \neq c_2 \in \mathbb{Q} \), and we employ a general method introduced in our article [DKY] to control the number of common zeroes of two distinct height functions on \( \mathbb{P}^1(\mathbb{Q}) \). The proofs differ from those of [DKY], in that here we use results from classical function theory and complex dynamics to obtain our desired estimates at the archimedean places of a number field.

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