Homework 1

1. **a**. Prove that open sets and closed sets are both G_δ and F_σ.

b. Is $\mathbb{Q}$ a G_δ set? Is it an F_σ set?

2. **a**. Prove that the function $f(x) = \begin{cases} 1/q & \text{if } x = p/q \text{ in lowest terms, } q > 0 \\ 0 & \text{if } x \text{ is irrational} \end{cases}$ is continuous precisely at the irrationals.

b. Prove that no function $f : \mathbb{R} \to \mathbb{R}$ can be continuous precisely at the rationals.

3. Royden, Section 2.2, Exercise 10.

4. Royden, Section 2.3, Exercise 15.

5. Royden, Section 2.4 Exercise 20.