Homework 3

1. If f is a function such that $f = \chi_{[0,1]}$ a.e., prove that f cannot be continuous on all $\mathbb{R}$.

2. If f is any measurable function, prove that there is a sequence $\{f_k\}$ of continuous functions, such that $f_k \to f$ a.e.

3. **a**. If x is any irrational, prove that there are infinitely many distinct rationals p/q such that

$$\left| x - \frac{p}{q} \right| \leq \frac{1}{q^2}$$

b. In contrast, if $\alpha > 2$, prove that the set of $x \in \mathbb{R}$ for which there are infinitely many distinct rationals p/q such that

$$\left| x - \frac{p}{q} \right| \leq \frac{1}{q^\alpha}$$

has measure 0.

4. Royden, Section 3.1, Exercise 7.

5. Royden, Section 3.2, Exercise 24.