Homework 7

1. Prove that L^∞ is complete.

2. Define $T : L^p([0,1]) \to L^p([0,1])$ by $T(f(x)) = xf(x)$ (here $1 \le p < \infty$).

    a. Prove that T has an operator norm, i.e., that there is a smallest number $||T||$ with $||T(f)||_p \le ||T||||f||_p$ for all $f \in L^p([0,1])$. What is $||T||$?

    b. However, prove that if $\bar{B}$ is the unit ball in $L^p([0,1])$, then $T(\bar{B})$ is not contained in any compact set.

3. Royden, Section 7.1, Exercise 4.

4. Royden, Section 7.2, Exercise 14.

5. Royden, Section 7.2, Exercise 18.