Math 331 discussion problems

TA: Alex Karapetyan

March 2, 2023

These are extra practice problems, not to be handed in.

1. For each R and M, determine whether M is free.

   (a) $R = \mathbb{C}[x,y]$, $M = (x,y)$.

   (b) $R = \mathbb{Z}$, $M = \mathbb{Z}[\sqrt{-5}]$.

   (c) $R = \mathbb{Z}$, $M = \mathbb{Q}$.

2. Suppose R is a commutative ring with $1 \neq 0$ such that every finitely generated R-module is free. Prove that R is a field.

3. Let R be a ring and let M be an abelian group. Prove that the structure of an R-module on M is equivalent to that of a ring homomorphism

$$R \longrightarrow \mathrm{End}(M)$$

where $\mathrm{End}(M)$ denotes the ring of abelian group homomorphisms $M \to M$.