

## Homework 6

1. Compute all homology groups of:

- $S^n$
- $T \vee T$
- $T \# T$  (genus 2 torus)

2. Let  $f : X \rightarrow Y$  be a simplicial map of  $\Delta$ -complexes and define  $f_n : C_n(X) \rightarrow C_n(Y)$  to be the linear map such that

$$f_n([v_0, \dots, v_n]) = \begin{cases} \text{sgn}(\sigma_n)[w_0, \dots, w_n] & \text{if } f(v_i) \neq f(v_j) \text{ for all } i \neq j \\ 0 & \text{otherwise} \end{cases}$$

where  $f(v_i) = w_{\sigma_n(i)}$  for all  $i$ . Prove that  $(f_n)$  is a chain map.

3. Suppose  $X$  is a finite  $\Delta$ -complex (i.e., it contains only finitely many simplices). Let  $c_n$  denote the number of  $n$ -simplices in  $X$  and define

$$\chi(X) = \sum_{n=0}^{\infty} (-1)^n c_n$$

- Prove that  $\chi(X) = \sum_{n=0}^{\infty} (-1)^n \text{rk} H_n(X)$ .
- Prove that if  $Y$  is another finite  $\Delta$ -complex then  $\chi(X \times Y) = \chi(X)\chi(Y)$ .