# Project 2

Due Monday, November 20.

In this project, you will model the suspension system of a car.

Consider a single wheel assembly as a mass $m$ (the corner of the car above the wheel) attached to a point (the wheel) via a spring and a damper (shock absorber). The spring has spring constant $k$ and the damping is proportional to the relative velocity (i.e., the difference between the velocities of the two ends of the damper) via the constant $b$; as usual $k, b > 0$. Let $h(t)$ denote the height of the ground below the wheel (with respect to some fixed level).

(a) If the car drives along flat ground (i.e., $h = 0$), derive an ODE for the displacement $y(t)$ of the mass at time $t$.

(b) Suppose we have a 1400 kg car (so each wheel assembly is 350 kg) with a spring constant of 1050 N/m and shock absorbers with damping coefficient 700 Ns/m. If the car begins at rest and drives on level ground, then if the suspension begins at equilibrium, it will of course remain at equilibrium. However, suppose instead that the spring is stretched by 10 cm at the start (for example, the car is dropped off a tow truck, causing the spring to stretch). Find $y$ and express your solution in the form $A(t)\cos(\omega_0 t - \theta)$.

(c) Suppose we replace the shock absorbers in the car from (b). Which damping coefficients produce underdamping, overdamping, and critical damping?

(d) (Mathematica) Graph solutions illustrating each of the different types of behavior corresponding to the level of damping. Assume the same initial conditions as in (b). Which level of damping should car manufacturers choose? Explain.

(e) Suppose now that the ground is not necessarily flat, i.e., that $h(t)$ is an arbitrary differentiable function. Determine the ODE satisfied by $y$.

(e) Explain why the long-term behavior of $y$ does not depend on the initial conditions.

(f) To model a bumpy road, it is common to use $h(t) = A\sin(\omega t)$. Set up an ODE for $y$ in this case.

(g) Suppose the car in (b) drives at a constant speed of 30 m/s on a bumpy road, as in (f), with successive peaks .25 m high and $30\pi$ m apart. Determine $y$ if the mass begins at equilibrium and at rest.

(h) (Mathematica) Graph several solutions to the equation in (g) under various initial conditions on the same set of axes. Observe the behavior predicted by your answer to (e).

(i) With the car from (b) on the road from (g), determine the speed that maximizes the amplitude of the long-term behavior of the system.

(j) (Mathematica) Graph the solution corresponding to your answer in (g) together with solutions corresponding to several other speeds, both faster and slower, on the same set of axes.