Project 3

Due Monday, December 4.

In this project, you will use delta functions to model the effect of an impulsive force.

(a) Consider a mass m attached to an undamped spring with spring constant k. Let y_0 denote the initial displacement of the mass and v_0 its initial velocity. Suppose that at time $t = c$, the mass is struck with a hammer, providing an impulse $A > 0$. Determine the displacement of the mass at time t.

(b) (Mathematica) Graph your solution to (a). Also graph the solutions obtained when the impulsive force is modeled by the functions denoted d_τ in the book for $\tau = .5, .1, .01$. You will need to choose values for all the constants (m, k, y_0, v_0, c, and A).

(c) Note that even after the mass is hit, it continues to oscillate (as you would expect from an undamped spring). However, show that it is always possible to choose the time $t = c$ and the amplitude A so that the mass stops vibrating after it is hit.

(d) (Mathematica) Graph the corresponding solution for the values of m, k, y_0, and v_0 you used in (b) and the values of c and A you found in (c).

(e) Now suppose the spring is damped, with damping coefficient b. With the same impulsive force (amplitude A at time $t = c$), determine the displacement of the mass at time t.

f) Show that for a damped spring if the mass begins at equilibrium and at rest, all solutions tend to equilibrium as $t \to \infty$, but it is *not* possible to choose c and A so as to stop the vibration of the mass after it is struck.

(g) Back to the undamped case: Suppose a unit mass on a spring with $k = 1$ begins at equilibrium and at rest. At each instant $t = 0, \pi, 2\pi, 3\pi, \ldots$, the mass is struck with a hammer, providing a unit impulse. Determine the displacement of the mass at time t. (You may assume that Laplace transforms of convergent infinite series may be computed term-by-term.)

(h) With the same setup as in (g), suppose instead the strikes are at $t = 0, 2\pi, 4\pi, \ldots$. Determine the displacement at time t.

(i) (Mathematica) Graph your solutions to (g) and (h).

(j) Describe the qualitative differences in the behavior of the system in (g) and the one in (h). Analyze these differences and explain them in physical terms.