Homework 4

Due Monday, Oct. 30.

1. Find the complex Fourier series for x^2 on $[-L, L]$ and show that it is equal to the real series we found in class.

2. Find the Fourier transform of $f(x) = \begin{cases} e^{-x} & \text{if} \quad x \geq 0 \\ \\ 0 & \text{if} \quad x < 0 \end{cases}$

3. Show that in spherical coordinates, the 3-dimensional Laplace equation becomes

$$u_{\rho\rho} + \frac{2}{\rho}u_\rho + \frac{1}{\rho^2}\left[u_{\phi\phi} + (\cot\phi)u_\phi + \frac{1}{\sin^2\phi}u_{\theta\theta}\right] = 0$$

4. Suppose u satisfies the 3-dimensional Laplace's equation with axial symmetry (i.e., u is independent of θ). If $u(\rho, \phi) = \Phi(\rho)\Psi(\phi)$ determine Φ.

5. Prove that the solution of the 1-dimensional heat equation on $[0, L]$

$$\begin{aligned} &u_t = \alpha^2 u_{xx} \\ &u(0,t) = T_1, u(L,t) = T_2 \\ &u(x,0) = f(x) \end{aligned}$$

is unique, as follows:

a) Suppose u_1, u_2 are two solutions and set $u = u_1 - u_2$. Show that

$$\frac{1}{2}\frac{\partial}{\partial t}(u^2) + \alpha^2 u_x^2 + \frac{\partial}{\partial x}(-\alpha^2 u u_x) = 0.$$

b) Integrate the above expression on $[0, L]$ and use the result to explain why $u = 0$.