Homework 5

Due Monday, Nov. 6.

1. Solve for u.

$$\begin{aligned} &u_t = \alpha^2 u_{xx} && t, x > 0 \\ &u_x(0,t) = 0 && t > 0 \\ &u(x,0) = f(x) && x > 0 \end{aligned}$$

(Extend f to the entire real line as an even function.)

2. Solve for u.

$$\begin{aligned} &u_{xx} + u_{yy} = 0 && y > 0 \\ &u(x,0) = f(x) \end{aligned}$$

Assuming f is real-valued, express your answer as a real-valued function.

3. a) Prove that

$$\int_{-\infty}^{\infty} \hat{f}(u)g(u)du = \int_{-\infty}^{\infty} f(u)\hat{g}(u)du$$

for all f, g for which both sides are defined.

b) Prove that

$$\int_{-\infty}^{\infty} |f(u)|^2 du = \int_{-\infty}^{\infty} |\hat{f}(u)|^2 du$$

(This is the Fourier transform analogue of Parseval's Identity. You may want to begin by showing that if $\bar{f}$ is the complex conjugate of f, then $\hat{\bar{f}}(s) = \bar{\hat{f}}(-s)$.)

4. Prove that $f * \delta = f$ for every f.

5. a) Solve the heat equation on $[-L, L]$.

$$\begin{aligned} &u_t = \alpha^2 u_{xx} && x \in [-L, L] \\ &u(-L,t) = u(L,t) \\ &u_x(-L,t) = u_x(L,t) \\ &u(x,0) = f(x) && x \in [-L, L] \end{aligned}$$

Express your solution in terms of the *complex* Fourier series for f.

b) By taking the limit of the function you found in a) as $L \to \infty$, show that you get the solution of the heat equation on $\mathbb{R}$ that we found in class.