Practice Problems

1. Find the Fourier transforms of the following functions.

   - $f(x) = e^{-|x|}$

   - $f(x) = \begin{cases} |x| & \text{if } |x| \le 1 \\ \\ 0 & \text{otherwise} \end{cases}$

   - $f(x) = \begin{cases} x^2 & \text{if } |x| \le 1 \\ \\ 0 & \text{otherwise} \end{cases}$

   - $f(x) = \begin{cases} 3x^2 & \text{if } |x| \le 10 \\ \\ 0 & \text{otherwise} \end{cases}$

   - $f(x) = \begin{cases} \frac{1}{12}x^2 & \text{if } 2 \le x \le 4 \\ \\ e^{\pi(x-2)} & \text{if } x < 2 \\ \\ 0 & \text{if } 4 > x \end{cases}$

2. Solve for u.

$$u_t = 2u_{xx} \qquad x < 0, t > 0$$
$$u_x(0, t) = 0 \qquad t > 0$$
$$u(x, 0) = e^{x+1} \qquad x < 0$$

3. Solve for u.

$$u_{tt} = u_{xx}$$
$$u(x, 0) = 0$$
$$u_t(x, 0) = e^{-x^2}$$

4. Solve for u.

$$u_t = u_{xx} + 4u_x$$
$$u(x, 0) = e^{-7|x|}$$