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Existence and the Singular Relaxation Limit for the Inviscid Hydrodynamic Energy Model

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Abstract

It is well known to particle physicists that scattering dominated transport processes lead to drift-diffusion regimes. In this paper, we give a mathematical study of this result, beginning with the full hydrodynamic model, and identifying scaled drift-diffusion as the zero relaxation limit. In order to provide a more complete background of drift-diffusion, we describe steady-state parameter regions in the introduction. This material is not essential for the paper.

1 Introduction

In this chapter, we shall study a hydrodynamic model for charged carrier transport, and its singular relaxation drift-diffusion limit. Although the emphasis of the chapter will be on the theory of the hydrodynamic model and its limit, and although the theory will proceed for the evolution system, we shall present in the introduction an idealized steady-state drift-diffusion model, so that the reader unfamiliar with this area can gain some appreciation for the rich and diverse behavior possible even in this special case. After presenting this model, we shall then discuss, in the remaining part of the introduction, the context in which the principal results of the paper are derived. Before proceeding to the steady-state model, we shall describe the setting of charge transport in the principal application areas of semiconductors and ionic channels.

It is helpful to view semiclassical semiconductor modeling as an hierarchical structure, in which the Boltzmann transport equation forms the summit, and the drift-diffusion model the base; intermediate are systems derived from moments of the Boltzmann transport equation. The hydrodynamic model is an example, and it offers the promise of simulation detail, as well as feasibility. It does have the drawback of requiring adequate closure assumptions and accurate

representations for the average collision mechanisms. In addition, the model has hyperbolic as well as parabolic components for the moment equations, in the transient case, and thus issues of shock capturing mechanisms arise. On the other hand, the drift-diffusion model has only parabolic components, exclusive of the Poisson equation, which must be adjoined to any model in the hierarchy. An interesting derivation of the drift-diffusion model from the Boltzmann transport equation, making use of the diffusion approximation, is carried out in [10]. This model is by far the best understood of the models. It was introduced by Van Roosbroeck in 1950 [28] as a conservation system for electron and hole carriers, with the ambient electric field determined from the Poisson equation. This model is closely related to the older (by sixty years) ionic transport model of Nernst and Planck, which is described in detail in [24]. It is of interest that the current-voltage curves in biological membrane channels are derived from mechanisms closely related to those described here, with similar systems of equations involved in the modeling. The reader is invited to consult [23] for scientific detail.

1.1 Idealized Drift–Diffusion

Physically, this is a case of constant temperature in which friction, caused by collisions, dominates. We select the case of *vanishing* permanent charge and *simple* boundary conditions. It describes:

- Electrons and holes in a pure crystalline lattice; or,
- Positive and negative ions in a vacuum environment,

together with equilibrium boundary conditions. The steady system is defined by continuity equations for

- (1) Electric potential ϕ ; charge flux: $-(\text{dielectric}) \times \phi'$;
- (2) Negative carrier (electron/anion) concentration n ;

$$\text{Charge: } -\bar{e}, \quad \text{Current: } J_n = \bar{e}D_n \left(\underbrace{n'}_{\text{diffusion}} \quad \underbrace{-n\phi'}_{\text{drift}} \right);$$

- (3) Positive carrier (hole/cation) concentration p ;

$$\text{Charge: } \bar{e}, \quad \text{Current: } J_p = \bar{e}D_p(p' + p\phi').$$

The equations (with vanishing permanent charge) are given by:

$$\epsilon^2 \phi'' - n + p = 0, \tag{1.1}$$

$$J_n = \text{const.}, \tag{1.2}$$

$$J_p = \text{const.}, \tag{1.3}$$

in $(0, 1)$, with boundary conditions,

$$p(0) = p_L > 0, \quad n(0) = n_L > 0, \tag{1.4}$$

$$p(1) = p_R > 0, \quad n(1) = n_R > 0, \tag{1.5}$$

$$\phi(0) = \phi_0, \quad \phi(1) = \phi_1. \tag{1.6}$$

Without loss of generality, take $\phi_1 = 0$. Above, D_n and D_p denote diffusion coefficients. We highlight here the following definitions:

- Potential difference: $V = \phi_0$,
- Physical current: $I = J_n + J_p$,
- Negative of (mass) flux: $J = J_n - J_p$.
- ϵ is the quotient of two lengths, intrinsic and extrinsic.

A notation in terms of dimensionless ratios is given by:

$$\rho_L = \frac{p_L}{n_L}, \quad \rho_R = \frac{p_R}{n_R}. \quad (1.7)$$

System (1.1)–(1.6) has *simple* boundary values if $\rho_L = \rho_R = 1$. In this case, denote the common values by c_L, c_R , respectively.

There are four modes for simple boundary conditions as is demonstrated in [22]:

Case 1. $c_L \leq c_R, V \geq 0$.

$$(1) IV \geq 0, \quad J \geq 0; \quad (2) \phi_x \leq 0; \quad (3) n \geq p; \quad (4) n_x, p_x \geq 0.$$

Case 2. $c_L \leq c_R, V \leq 0$.

$$(1) IV \geq 0, \quad J \geq 0; \quad (2) \phi_x \geq 0; \quad (3) n \leq p; \quad (4) n_x, p_x \geq 0.$$

Case 3. $c_L \geq c_R, V \leq 0$.

$$(1) IV \geq 0, \quad J \leq 0; \quad (2) \phi_x \geq 0; \quad (3) n \geq p; \quad (4) n_x, p_x \leq 0.$$

Case 4. $c_L \geq c_R, V \geq 0$.

$$(1) IV \geq 0, \quad J \leq 0; \quad (2) \phi_x \leq 0; \quad (3) n \leq p; \quad (4) n_x, p_x \leq 0.$$

Fig. 1 illustrates a typical case. In all cases, the solution is unique, as demonstrated in [22].

1.2 The Hydrodynamic Model

This model may alternately be defined from a microscopic starting point, the semiclassical Boltzmann equation, in which moments are computed with respect to group velocity, and the system closed at three equations with constitutive relations among the averaged quantities, or it can be derived from a macroscopic starting point, in which the system then becomes a conservation law system for a charged fluid, incorporating particle number, momentum, and energy balance equations. Both forms of derivation are carried out in the book [14] Chap. 2. The result is the following system, with a copy for each charge species:

FIG. 1. The electric potential profile and concentration profiles for the simple boundary condition, Case 1. Dimensionless concentration and potential units employed. Reproduced from [2].

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho v) = C_\rho, \\ \partial_t p + v(\nabla \cdot p) + (p \cdot \nabla)v = -\frac{\bar{e}}{\bar{m}} \rho F - \nabla(\rho k_b T / \bar{m}) + C_p, \\ \partial_t E + \nabla \cdot (v E) = -\frac{\bar{e}}{\bar{m}} \rho v \cdot F - \nabla \cdot (v \rho k_b T / \bar{m}) + \nabla \cdot (\kappa \nabla T) + C_E. \end{cases} \quad (1.8)$$

Here, ρ denotes particle mass density, related to concentration n and effective mass \bar{m} via $\rho = \bar{m}n$, p denotes particle momentum density, related to velocity v through $p = \rho v$, and E the mechanical energy density. F denotes the electric field, T the carrier temperature, κ the heat conductivity, k_b Boltzmann's constant, and C_ρ , C_p , and C_E denote relaxation expressions. The systems are coupled through the Poisson electrostatic equation as well. The momentum relaxation time τ_p is given via

$$\frac{p}{\tau_p} := -C_p.$$

The energy relaxation time τ_w is given via

$$-\frac{W - W_0}{\tau_w} := C_W.$$

Here, W_0 denotes the rest energy, $\frac{3}{2}nk_b\bar{T}$, where \bar{T} is the lattice temperature. Note that the relaxation time reciprocals appearing here have the units of frequencies, and suggest time scales over which collisions are likely to occur in influencing momentum or energy. The forms for the relaxation times used in [1]

on the basis of higher order moments, are:

$$\begin{aligned}\tau_p &= c_p/T, \\ \tau_w &= c_w \frac{T}{T + \bar{T}} + \frac{1}{2}\tau_p.\end{aligned}$$

Here, c_p and c_w are physical constants. The hydrodynamic model was first derived by Bløtekjær [3] for semiconductor devices. It can describe multiple species of charge carriers (e.g., upper and lower valley electrons and holes) by adding extra copies of (1.8). This model also governs an important biological process in which ions move into biological cells through pores in proteins that are holes in dielectrics [5]. The model (1.8) plays an important role in simulating the behavior of charged carriers in submicron semiconductor devices; some of its computational and physical aspects have been discussed in [11, 25, 16]. The existence of solutions and the convergence of Newton's method to the steady-state subsonic electron flow have been proved in [13]. Also, an electron shock wave in the case of steady-state for (1.8) for submicron semiconductor devices was first simulated in [12].

1.3 Regimes Defined by Damping

We identify three essential regimes defined by the hydrodynamic model:

- (1) The full hydrodynamic regime;
- (2) A regime in which inertial effects are negligible over the momentum relaxation time scale;
- (3) A regime, contained within the preceding, in which temperature variation is negligible, so that heat flux is insignificant. This corresponds to the drift-diffusion regime. It is characterized as friction dominated.

In the first regime, friction is not dominant. In the second, resistance to momentum transitions is significant over the relaxation time scale. In the third regime, additional resistance to heat flow (temperature gradients) is significant. Regime two may be defined via a critical limit of the hydrodynamic model, as we now explain. The first and third equations in (1.8) remain as in the hydrodynamic model. However, the momentum equation is rewritten, and the limit,

$$\tau_p \rightarrow 0,$$

is taken to obtain a constitutive relation for the current, given by:

$$J = \frac{\bar{e}D}{\bar{m}} \left[\frac{T}{\bar{T}} \nabla \rho + \rho \nabla \left(\frac{T}{\bar{T}} - \frac{\bar{e}}{k_b \bar{T}} \phi \right) \right]. \quad (1.9)$$

Here, D is the diffusion coefficient and ϕ is the electrostatic potential. This is the interpretation of the inertial approximation. The third regime follows from the second when isothermal conditions prevail.

These are strictly heuristic considerations. It is the purpose of this paper to derive the third regime in a completely rigorous fashion, as a scaled limit of the

one carrier model as τ_p and τ_w tend to zero in a controlled manner. In fact, the fundamental inequality relating τ_p and τ_w , and required by the theory developed below, is satisfied by the representations given above.

We close the introduction by exhibiting the results of a series of simulations carried out by decreasing τ_w via increasing a parameter, the saturation velocity, v_s , which is related to c_w in an inverse square fashion. The application is to the biological regime, that of ionic channels. The tendency of regime (1) toward regime (3) is evident in these pictures.

FIG. 2. The solid curve gives the temperature for $v_s = 5 \times 10^{-6}$, the dotted curve is for $v_s = 10^{-5}$, the short-dashed curve is for $v_s = 2 \times 10^{-5}$, and the long-dashed curve is for $v_s = 5 \times 10^{-5}$. The decrease of temperature coincides with efficient damping of energy exchange. The figure is taken from [5]. The units of v_s are $\mu m/ps$.

2 Energy Method for a Scalar Equation with Damping

In this section, through a scalar model with damping, we present an efficient method, the so-called energy method, for determining the global existence, uniqueness, and asymptotic behavior of classical smooth solutions for nonlinear partial differential equations with certain dissipation. Such a method has been successfully applied to problems of classical smooth solutions for nonlinear wave equations in [17], nonlinear hyperbolic Volterra integrodifferential equations in [9], nonlinear thermoelasticity in [27], as well as for the compressible Navier-Stokes equations in [21] and [6] (see also the references cited in [17, 9, 27, 21, 6]).

Consider the following scalar model with damping:

$$u_t + f(u)_x + u = 0, \tag{2.1}$$

FIG. 3. The IV curves are calculated for different values of v_s and compared to the drift-diffusion (PNP) case. The figure is taken from [5].

with Cauchy data

$$u|_{t=0} = u_0(x). \quad (2.2)$$

The local existence and uniqueness of the smooth solution $u(x, t)$ for the Cauchy problem (2.1)–(2.2) can be found in [19]. It can be obtained by using the standard Contraction Mapping Theorem of Banach (see Section 3.1 for such an application for the inviscid hydrodynamic energy model). In this section, we present the energy method, through the scalar model (2.1), in order to motivate the more general approach involved in establishing the global existence, uniqueness, and asymptotic behavior of classical smooth solutions to systems of conservation laws to follow in the next section. It is particularly intended for readers not familiar with the techniques of this subject.

Theorem 1 *Assume that initial data $u_0(x) \in H^2(\mathbf{R})$. Then there exists a unique global smooth solution of the problem (2.1)–(2.2) satisfying*

$$u(x, t) \rightarrow 0, \quad \text{uniformly when } t \rightarrow \infty.$$

Proof It suffices to obtain the energy estimates for the local solution $u(x, t)$. Now we derive such estimates in considerable detail, as an illustration for the following Sections 3 and 4, for the smooth solution $u(x, t) \in H^2((-\infty, \infty) \times [0, t_0])$. In the course of these estimates, we make certain assumptions on f , not always made explicit. We first assume *a priori* that

$$\|u\|_{C^1(\mathbf{R} \times [0, t_0])} \leq \varepsilon, \quad (2.3)$$

for sufficiently small ε . Then we prove that there exists $\varepsilon_0 = \varepsilon_0(\varepsilon, f)$ such that,

when

$$\|u_0\|_{H^2} \leq \varepsilon_0,$$

the *a priori* assumption (2.3) is indeed achieved.

Multiplying (2.1) by u , one has

$$\left(\frac{u^2}{2}\right)_t + u^2 + \left(\int_0^u v f'(v) dv\right)_x = 0.$$

Integrating over $\mathbf{R} \times [0, t)$, $t \leq t_0$ leads to

$$\int_{-\infty}^{\infty} u^2(x, t) dx + 2 \int_0^t \int_{-\infty}^{\infty} u^2(x, s) dx ds = \int_{-\infty}^{\infty} u_0^2(x) dx. \quad (2.4)$$

Differentiating (2.1) with respect to x and then multiplying by u_x , one has

$$\left(\frac{u_x^2}{2}\right)_t + u_x^2 + \frac{1}{2} f''(u) u_x^3 = -\frac{1}{2} (f'(u) u_x^2)_x.$$

Integrating over $\mathbf{R} \times [0, t)$, $t \leq t_0$ leads to

$$\begin{aligned} & \int_{-\infty}^{\infty} u_x^2(x, t) dx + 2 \int_0^t \int_{-\infty}^{\infty} u_x^2(x, s) dx ds \\ & \leq \int_{-\infty}^{\infty} u_{0x}^2(x) dx + C \int_0^t \int_{-\infty}^{\infty} |u_x|^3(x, s) dx ds. \end{aligned} \quad (2.5)$$

Differentiating (2.1) with respect to t and then multiplying by u_t , one has

$$\left(\frac{u_t^2}{2}\right)_t + u_t^2 + \frac{1}{2} f''(u) u_x u_t^2 = -\frac{1}{2} (f'(u) u_t^2)_x.$$

Integrating over $\mathbf{R} \times [0, t)$, $t \leq t_0$ leads to

$$\begin{aligned} & \int_{-\infty}^{\infty} u_t^2(x, t) dx + 2 \int_0^t \int_{-\infty}^{\infty} u_t^2(x, s) dx ds \\ & \leq \int_{-\infty}^{\infty} u_{0t}^2(x) dx + C \int_0^t \int_{-\infty}^{\infty} (|u_x| u_t^2)(x, s) dx ds \\ & \leq C \int_{-\infty}^{\infty} (u_0^2 + u_{0x}^2)(x) dx + C \int_0^t \int_{-\infty}^{\infty} (|u_x| u_t^2)(x, s) dx ds. \end{aligned} \quad (2.6)$$

Here we have used the notation $u_{0t}(x) = u_t(x, 0)$, and estimated this quantity by (2.1). Differentiating (2.1) with respect to xx and then multiplying by u_{xx} , one has

$$\left(\frac{u_{xx}^2}{2}\right)_t + u_{xx}^2 + 2f''(u) u_x u_{xx}^2 + f'''(u) u_x^3 u_{xx} = -\frac{1}{2} (f'(u) u_{xx}^2)_x.$$

Integrating over $\mathbf{R} \times [0, t)$, $t \leq t_0$ leads to

$$\begin{aligned}
& \int_{-\infty}^{\infty} u_{xx}^2(x, t) dx + 2 \int_0^t \int_{-\infty}^{\infty} u_{xx}^2(x, s) dx ds \\
& \leq \int_{-\infty}^{\infty} u_{0xx}^2(x) dx + C \int_0^t \int_{-\infty}^{\infty} (|u_x u_{xx}^2| + |u_x^3 u_{xx}|)(x, s) dx ds \\
& \leq C \int_{-\infty}^{\infty} u_{0xx}^2(x) dx + C \int_0^t \int_{-\infty}^{\infty} (|u_x u_{xx}^2| + u_x^6)(x, s) dx ds, \quad (2.7)
\end{aligned}$$

where we used the inequality,

$$ab \leq \delta a^2 + b^2/(4\delta). \quad (2.8)$$

The left hand side of (2.7) is understood to be the expression two lines above; for compactness, we absorbed the estimation term δu_{xx}^2 , and did not rewrite this left hand side. We employ this technique throughout the proof.

Differentiating (2.1) with respect to xt and then multiplying by u_{xt} , one has

$$\left(\frac{u_{xt}^2}{2}\right)_t + u_{xt}^2 + \frac{3}{2}f''(u)u_x u_{xt}^2 + f''(u)u_t u_{xx} u_{xt} + f'''(u)u_x^2 u_t u_{xt} = -\frac{1}{2}(f'(u)u_{xt}^2)_x.$$

Integrating over $\mathbf{R} \times [0, t)$, $t \leq t_0$ leads to

$$\begin{aligned}
& \int_{-\infty}^{\infty} u_{xt}^2(x, t) dx + 2 \int_0^t \int_{-\infty}^{\infty} u_{xt}^2(x, s) dx ds \\
& \leq \int_{-\infty}^{\infty} u_{0xt}^2(x) dx + C \int_0^t \int_{-\infty}^{\infty} (|u_x u_{xt}^2| + |u_t u_{xx} u_{xt}| + |u_x^2 u_t u_{xt}|)(x, s) dx ds \\
& \leq C \int_{-\infty}^{\infty} (u_{0x}^2 + u_{0x}^4 + u_{0xx}^2)(x) dx \\
& \quad + C \int_0^t \int_{-\infty}^{\infty} (|u_x u_{xt}^2| + u_t^2 u_{xx}^2 + u_x^4 u_t^2)(x, s) dx ds, \quad (2.9)
\end{aligned}$$

where we used the inequality (2.8) and (2.1), and the notation, $u_{0xt} = u_{xt}(x, 0)$.

Similarly, differentiating (2.1) with respect to tt and then multiplying by u_{xt} , one has

$$\left(\frac{u_{tt}^2}{2}\right)_t + u_{tt}^2 + \frac{1}{2}f''(u)u_x u_{tt}^2 + 2f''(u)u_t u_{xt} u_{tt} + f'''(u)u_x u_t^2 u_{tt} = -\frac{1}{2}(f'(u)u_{tt}^2)_x.$$

Integrating over $\mathbf{R} \times [0, t)$, $t \leq t_0$ leads to

$$\begin{aligned}
& \int_{-\infty}^{\infty} u_{tt}^2(x, t) dx + 2 \int_0^t \int_{-\infty}^{\infty} u_{tt}^2(x, s) dx ds \\
& \leq \int_{-\infty}^{\infty} u_{0tt}^2(x) dx + C \int_0^t \int_{-\infty}^{\infty} (|u_x u_{tt}^2| + |u_t u_{xt} u_{tt}| + |u_x u_t^2 u_{tt}|)(x, s) dx ds
\end{aligned}$$

$$\begin{aligned}
&\leq C \int_{-\infty}^{\infty} (u_{0xx}^2 + u_{0x}^4 + u_0^4 + u_{0x}^2 + u_0^2)(x) dx \\
&\quad + C \int_0^t \int_{-\infty}^{\infty} (|u_x u_{tt}^2| + u_t^2 u_{xt}^2 + u_x^2 u_t^4)(x, s) dx ds, \tag{2.10}
\end{aligned}$$

where we used the inequality (2.8) and (2.1), and the notation, $u_{0tt} = u_{tt}(x, 0)$.

Adding (2.4)–(2.7) and (2.8)–(2.10), one has

$$\begin{aligned}
&\int_{-\infty}^{\infty} (u^2 + u_x^2 + u_t^2 + u_{xx}^2 + u_{xt}^2 + u_{tt}^2)(x, t) dx \\
&\quad + 2 \int_0^t \int_{-\infty}^{\infty} (u^2 + u_x^2 + u_t^2 + u_{xx}^2 + u_{xt}^2 + u_{tt}^2)(x, s) dx ds \\
&\leq C \int_{-\infty}^{\infty} (u_0^2 + u_0^4 + u_{0x}^2 + u_{0x}^4 + u_{0xx}^2)(x) dx \\
&\quad + C \int_0^t \int_{-\infty}^{\infty} [u_x^2 u_t^2 (u_t^2 + u_x^2) + u_x^6 + |u_x| (u_x^2 + u_t^2 + u_{xx}^2 + u_{xt}^2 + u_{tt}^2) \\
&\quad\quad + u_t^2 (u_{xx}^2 + u_{xt}^2)](x, s) dx ds \\
&\leq C \|u_0\|_{H^2(\mathbf{R})}^2 + C\varepsilon \int_0^t \int_{-\infty}^{\infty} (u_x^2 + u_t^2 + u_{xx}^2 + u_{xt}^2 + u_{tt}^2) dx ds. \tag{2.11}
\end{aligned}$$

For sufficiently small $\varepsilon > 0$, one has from (2.11) that

$$\begin{aligned}
[N(t)]^2 &= \int_{-\infty}^{\infty} (u^2 + u_x^2 + u_t^2 + u_{xx}^2 + u_{xt}^2 + u_{tt}^2)(x, t) dx \\
&\quad + 2 \int_0^t \int_{-\infty}^{\infty} (u^2 + u_x^2 + u_t^2 + u_{xx}^2 + u_{xt}^2 + u_{tt}^2)(x, s) dx ds \\
&\leq C \|u_0\|_{H^2(\mathbf{R})}^2. \tag{2.12}
\end{aligned}$$

Notice that C does not depend upon ε if the latter is sufficiently small. From the local existence and uniqueness theorem, there exists an interval $[0, t_0)$ such that (2.1) has a unique smooth solution. It follows from the Sobolev inequality that

$$\|u\|_{C^1(\mathbf{R} \times [0, t_0])} \leq \Gamma \sup_{0 \leq t \leq t_0} N(t),$$

where Γ is a positive constant. Choose ε_0 such that

$$\varepsilon_0 \leq \frac{\varepsilon}{\Gamma \sqrt{C}}.$$

Then the estimate (2.12) implies

$$N(t) \leq \varepsilon / \Gamma,$$

as we assume *a priori*, which implies (2.3). By a technique developed fully in the next sections, we can apply the above arguments again, and continue the

local solution to the global solution in $[0, \infty)$ by use of the estimate (2.12). Alternatively, if one wishes to use only the information already derived, one can assume that continuation has been carried out maximally, and then use the C^1 estimates to continue locally from classical conservation law principles. This contradiction ensures the global continuation.

Therefore, we have proved the existence of the global smooth solution u satisfying

$$u \in L^2([0, \infty); H^2(\mathbf{R})); \quad \|u\|_{C(\mathbf{R})}(t) \rightarrow 0, \quad \text{when } t \rightarrow \infty.$$

This completes the proof. \square

3 Global Smooth Solutions of the Inviscid Hydrodynamic Energy Model

In this section, we consider the inviscid hydrodynamic energy model, rewritten from (1.8) as:

$$\rho_t + (\rho v)_x = 0, \quad (3.1)$$

$$(\overline{m}\rho v)_t + (\overline{m}\rho v^2 + \rho T)_x = \overline{e}\rho\phi_x - \frac{\overline{m}\rho v}{\tau_p}, \quad (3.2)$$

$$E_t + (vE + v\rho T)_x - (\kappa T_x)_x = \overline{e}\rho v\phi_x - \frac{E - \frac{3}{2}\rho\overline{T}}{\tau_w}, \quad (3.3)$$

$$\phi_{xx} = \overline{e}(\rho - d(x)), \quad (3.4)$$

where $\rho(x, t) > 0$, $v(x, t)$, $T(x, t)$, $\phi(x, t)$, $\kappa > 0$, $d(x, t) \geq d_0 > 0$, $\overline{e} > 0$, \overline{T} , and \overline{m} respectively denote *electron density*, *velocity*, *temperature* in energy units (the Boltzmann constant k_b has been set to 1), *electrostatic potential* in charge units per length (the dielectric has been set to 1), *thermal conductivity*, *doping profile* (background ion density), *electron charge*, *ambient device temperature*, and *effective electron mass*, and

$$E = \frac{3}{2}\rho T + \frac{1}{2}\overline{m}\rho v^2,$$

is the *energy density* for parabolic energy bands. The collision terms have been treated classically through momentum and energy relaxation times $\tau_p > 0$ and $\tau_w > 0$ (see the discussion in Section 1.2).

The existence of a global classical solution of the system (3.1)–(3.4) with additional viscosity was proved in [30]. However, for the Cauchy problem of (3.1)–(3.4), with large smooth initial data, it is proved in [29] that the solution generally develops a singularity, shock waves, and hence no global classical solution exists. The next question is whether there exists a global classical solution of (3.1)–(3.4), provided that the initial data are small, although the system (3.1)–(3.4) does not have the viscosity term as in [30]. In this section, we address this problem with the aid of the energy method as shown in Section 2, and prove that

there exists a global smooth solution of the inviscid system (3.1)–(3.4), provided that the initial data are small.

Consider the following initial-boundary value problem with impermeable insulated boundaries for (3.1)–(3.4):

$$\begin{aligned} (\rho, v, T)(x, 0) &= (\rho_0(x), \tau_p v_0(x), \bar{T} + \tau_w(T_0(x) - \bar{T})), & 0 \leq x \leq 1, \\ (v, T_x, \phi)(i, t) &= (0, 0, 0), \quad i = 0, 1, \quad t \geq 0, \end{aligned} \quad (3.5)$$

Such an initial condition is natural for the drift-diffusion limit (see Section 4). For the existence problem, this initial condition can be relaxed by modifying our energy estimates. Note that the boundary conditions for ϕ are those of electric neutrality.

In this section we use the energy method to prove the global existence, uniqueness, and asymptotic decay of classical smooth solutions for the problem (3.1)–(3.5) without the viscosity terms. This indicates that the relaxation term prevents the development of shock waves for smooth initial data with small oscillation.

We remark that, in the hydrodynamic equations (3.1)–(3.4), we can assume $\bar{m} = \bar{e} = 1$ without loss of generality. Otherwise we can rescale

$$(T, \bar{T}, \phi) \rightarrow (T/\bar{m}, \bar{T}/\bar{m}, \bar{e}\phi/\bar{m}),$$

in the first three equations and the resultant constant in the fourth equation will not add any analytical difficulty, and hence can be assumed to equal one in our analysis.

Using the Lagrangian coordinate (y, t) as used in [6]: $y = \int_0^x \rho(\xi, t) d\xi$, we obtain the full hydrodynamic model in Lagrangian coordinates:

$$(1/\rho)_t - v_y = 0, \quad (3.6)$$

$$v_t + (\rho T)_y = \rho \phi_y - v/\tau_p, \quad (3.7)$$

$$T_t - \frac{2}{3}(\kappa \rho T_y)_y + \frac{2}{3}\rho T v_y = \frac{2\tau_w - \tau_p}{3\tau_p \tau_w} v^2 - \frac{T - \bar{T}}{\tau_w}, \quad (3.8)$$

$$(\rho \phi_y)_y = 1 - \tilde{d}/\rho, \quad (3.9)$$

with the same initial-boundary conditions:

$$\begin{aligned} (\rho, v, T)(y, 0) &= (\rho_0(x(y, 0)), \tau_p v_0(x(y, 0)), \bar{T} + \tau_w(T_0(x(y, 0)) - \bar{T})), \\ (v, T_y, \phi)(i, t) &= (0, 0, 0), \quad i = 0, 1, \quad t \geq 0, \end{aligned} \quad (3.10)$$

where $\tilde{d}(y, t) = d(x(y, t))$.

Let

$$1/\rho - 1 = w_y, \quad v = w_t, \quad \theta = T - \bar{T}. \quad (3.11)$$

Then, the system (3.6)–(3.9) is transformed into the following form:

$$w_{tt} - \frac{\theta + \bar{T}}{(1+w_y)^2} w_{yy} + \frac{1}{1+w_y} \theta_y + \frac{w_t}{\tau_p} = \int_0^1 \int_\eta^y ((1 - \tilde{d}) - \tilde{d} w_y) d\xi d\eta, \quad (3.12)$$

$$\theta_t - \frac{2\kappa}{3(1+w_y)}\theta_{yy} + \frac{2\kappa\theta_y}{3(1+w_y)^2}w_{yy} + \frac{2(\theta+\bar{T})}{3(1+w_y)}w_{yt} + \frac{\theta}{\tau_w} - \frac{(2\tau_w-\tau_p)w_t^2}{3\tau_p\tau_w} = 0, \quad (3.13)$$

with the following initial-boundary conditions:

$$(w, w_t, \theta)(y, 0) = \left(\int_0^y \frac{1 - \rho_0(x(\xi, 0))}{\rho_0(x(\xi, 0))} d\xi, \tau_p v_0(x(y, 0)), \bar{T} + \tau_w (T_0(x(y, 0)) - \bar{T}) \right), \quad (3.14)$$

$$(w, \theta_y)(i, t) = (0, 0), \quad i = 0, 1, \quad t \geq 0. \quad (3.15)$$

In the following context, we use the following notations for simplicity:

$$\begin{aligned} \partial &= \partial^1 = (\partial_t, \partial_y), & \partial^2 &= (\partial_{tt}^2, \partial_{yt}^2, \partial_{yy}^2), & \partial^3 &= (\partial_{ttt}^3, \partial_{ytt}^3, \partial_{yyt}^3, \partial_{yyy}^3), \\ D &= D^1 = (\partial^0, \partial^1), & D^2 &= (\partial^0, \partial^1, \partial^2), & D^3 &= (\partial^0, \partial^1, \partial^2, \partial^3), \\ |\partial^0 u|^2 &= |u|^2, & |\partial^1 u|^2 &= |\partial_t u|^2 + |\partial_y u|^2, & |\partial^2 u|^2 &= |\partial_{tt} u|^2 + |\partial_{ty} u|^2 + |\partial_{yy} u|^2, \\ |\partial^3 u|^2 &= |\partial_{ttt} u|^2 + |\partial_{tty} u|^2 + |\partial_{tyy} u|^2 + |\partial_{yyy} u|^2, \\ |D^m u|^2 &= \sum_{j=0}^m |\partial^j u|^2, & m &= 1, 2, 3, \end{aligned}$$

and $C > 0$ is a universal constant and $C(N) > 0$ is a universal constant depending only on N .

3.1 Local Solutions

To establish the local existence for the problem (3.1)–(3.5), we apply Banach's contraction mapping theorem. For this purpose, we first consider the following linear problem:

$$w_{tt} - a_1(y, t)w_{yy} + a_3(y, t)\theta_y + w_t/\tau_p = f, \quad (3.16)$$

$$\theta_t - a_2(y, t)\theta_{yy} + a_4(y, t)w_{yy} + a_5(y, t)w_{yt} + a_6(y, t)w_t + \theta/\tau_w = 0, \quad (3.17)$$

$$(w, w_t, \theta)(y, 0) = (w_0, w_1, \theta_0)(y), \quad (w, \theta_y)(i, t) = (0, 0), \quad i = 0, 1. \quad (3.18)$$

Lemma 1 For $t_0 > 0$ and $1 \leq j \leq 6$, let

$$a_1, a_2 > 0, \quad a_j(y, t) \in C^1([0, 1] \times [0, t_0]), \quad \partial^2 a_j(y, t) \in L^\infty([0, t_0]; L^2(0, 1)).$$

Assume that the initial data satisfy

$$(w_{0y}, w_1)(y) \in H^2(0, 1), \quad \theta_0(y) \in H^3(0, 1); \quad (w_0, w_1, \theta_{0y})(i) = (0, 0, 0), \quad i = 0, 1.$$

Then, (3.16)–(3.18) has a unique smooth solution (w, θ) satisfying:

$$\begin{aligned} w(y, t) &\in C^2([0, 1] \times [0, t_0]), & (\theta, \partial\theta_y)(y, t) &\in C^1([0, 1] \times [0, t_0]), \\ \theta_{tt}(y, t) &\in L^\infty([0, t_0]; C[0, 1]), & (D^3 w, D^2 \theta, \partial^2 \theta_y)(y, t) &\in L^\infty([0, t_0]; L^2(0, 1)). \end{aligned}$$

Lemma 1 can be proved by applying the method of Faedo-Galerkin-Lions. For details, see Section 8.2 of Chapter 3 in [18]. One could also use Rothe's method of horizontal lines.

To establish the local existence of the initial-boundary problem (3.12)–(3.15), we modify the system (3.12)–(3.13) to avoid possible singular coefficients. Let $a(w_y)$ be a smooth function so that $a(w_y) = \frac{1}{1+w_y}$ for $|w_y| \leq \frac{1}{2}$ and $\frac{2}{3} \leq a(w_y) \leq 2$ for $|w_y| < \infty$. We also assume that $\theta + \bar{T} \geq \frac{\bar{T}}{2}$. This will cause no difficulty since we will have an *a posteriori* estimate with small $|w_y(y, t)|$ for small $t_0 > 0$.

Theorem 2 *Assume that initial data satisfy*

$$(\rho_0(y) - 1, v_0(y)) \in H^2(0, 1), \quad \theta_0(y) \in H^3(0, 1); \quad (v_0, \theta_{0y})(i) = (0, 0), \quad i = 0, 1.$$

Then, the problem (3.12)–(3.15) has a unique smooth solution (w, θ) satisfying

$$\begin{aligned} w(y, t) &\in C^2([0, 1] \times [0, t^*)), \quad (\theta, \partial\theta_y)(y, t) \in C^1([0, 1] \times [0, t^*)), \\ (D^3w, D^2\theta)(y, t) &\in L^\infty([0, t^*); L^2(0, 1)), \quad (\partial^2\theta_y)(y, t) \in L^2([0, t^*); L^2(0, 1)), \end{aligned}$$

defined on a maximal interval of existence $[0, t^)$. If $t^* < \infty$, then*

$$\int_0^1 |(D^3w, D^2\theta)|^2 dy + \int_0^t \int_0^1 |\partial^2\theta_y|^2 dy ds \rightarrow \infty, \quad t \rightarrow t^*. \quad (3.19)$$

Proof Step 1. For positive constants t_0 and N with

$$N^2 \geq 2\{\|(w_0, \theta)\|_{H^3(0,1)}^2 + \|w_{0t}\|_{H^2(0,1)}^2\},$$

we define

$$X(N, t_0) = \left\{ \begin{array}{l} (U, \Theta)(y, t) : \|(U, \Theta)\|_{X(N,t)} \leq N, \\ (U, U_t, \Theta)(y, 0) = (w_0, \tau_p v_0, \bar{T} + \tau_w(T_0 - \bar{T}))(y), \\ (U, \Theta_y)(i, t) = (0, 0), \quad 0 \leq t \leq t_0, \quad i = 0, 1, \end{array} \right\}$$

where

$$\|(U, \Theta)\|_{X(N,t)}^2 = \sup_{0 \leq t \leq t_0} \int_0^1 |(D^3U, D^2\Theta)|^2 dy + \int_0^t \int_0^1 |\partial^2\Theta_y|^2 dy ds.$$

Let $(U, \Theta) \in X(N, t_0)$. Consider the following linear system (3.16)–(3.17) with initial-boundary data,

$$(w, w_t, \theta)(y, 0) = (w_0, \tau_p v_0, \bar{T} + \tau_w(T_0 - \bar{T}))(y), \quad 0 \leq y \leq 1, \quad (3.20)$$

$$(w, \theta_y)(i, t) = (0, 0), \quad 0 \leq t \leq t_0, \quad i = 0, 1, \quad (3.21)$$

where

$$\begin{aligned} a_1(y, t) &= (\Theta + \bar{T})a(U_y), \quad a_2(y, t) = 2\kappa a(U_y)/3, \quad a_3(y, t) = a(U_y), \\ a_4(y, t) &= \frac{2\kappa\Theta_y}{3}a^2(U_y), \quad a_5(y, t) = \frac{2(\Theta + \bar{T})}{3}a(U_y), \quad a_6(y, t) = \frac{(\tau_p - 2\tau_w)U_t}{3\tau_p\tau_w}. \end{aligned} \quad (3.22)$$

All coefficients of (3.22) in the equations (3.16)–(3.17) satisfy the conditions

of Lemma 1. Thus, for every $(U, \Theta) \in X(N, t_0)$, the problem (3.16)–(3.17) and (3.20)–(3.21) has a unique solution $(w, \theta) \in X(M, t_0)$ for certain $M \geq N > 0$. Now we define a map F by solving (3.16)–(3.17) and (3.20)–(3.21), i.e., $(w, \theta) = F(U, \Theta)$.

Step 2. To show that there exists $\tilde{t} \leq t_0$ such that F maps $X(N, \tilde{t})$ into itself, we need *a priori* estimates. Since $(w, \theta) \in X(M, t_0)$, $M \geq N > 0$, it suffices to prove that there exists $\tilde{t} \leq t_0$ such that $(w, \theta) \in X(N, \tilde{t})$.

Multiply (3.16) and (3.17) by w_t and θ , respectively. Differentiate (3.16), (3.17) with respect to t and multiply by w_{tt} , θ_t , respectively. Differentiate (3.16), (3.17) with respect to y and multiply by w_{yt} , θ_y , respectively. Then we differentiate (3.16), (3.17) with respect to tt and multiply by w_{ttt} , θ_{tt} , respectively. Also we take second derivatives of (3.16) and (3.17) with respect to yt and multiply by w_{ytt} , θ_{yt} , respectively; and take derivatives of (3.16) and (3.17) with respect to yy and y , and multiply by $a_1 w_{yyt} + f$, θ_{yyy} , respectively. Finally we take derivatives of (3.16) with respect to yy and multiply by $(a_1 w_{yyt} + f)_t$. We then use integration by parts and the elementary inequality: $\alpha\beta \leq \frac{1}{2}(\alpha^2 + \beta^2)$, to obtain the following energy inequality:

$$\begin{aligned} & \frac{1}{2} \int_0^1 \left[w_t^2 + a_1 |D^2 w_y|^2 + |D^2 w_t|^2 + |D^2 \theta|^2 \right] dy \\ & + \int_0^t \int_0^1 \left[\frac{1}{\tau_p} |D^2 w_t|^2 + \frac{1}{\tau_w} |D^2 \theta|^2 + a_2 |D^2 \theta_y|^2 \right] dy ds \\ & \leq I(0) + C \int_0^t \int_0^1 \left[|D(Dw, \theta)| \left(\sum_{j=1}^6 |\partial^2 a_j| + |\partial a_4| \right) |D^2(Dw, \theta, \theta_y)| + |D^2 f \cdot D^2 w_t| \right. \\ & \quad \left. + \left(\sum_{j \neq 4} (|a_j| + |\partial a_j|) + |a_4| \right) |D^2(Dw, \theta, \theta_y)|^2 \right] dy ds, \end{aligned} \quad (3.23)$$

where $I(0)$ is the value of the first line above, in (3.23), evaluated at $t = 0$.

Since $(U, \Theta) \in X(N, t_0)$ and $(w, \theta) \in X(M, t_0)$, it follows from Sobolev's inequality that

$$\|(D^2 U, D\Theta)\|_{C([0,1] \times [0,t_0])} \leq CN, \quad \|(D^2 w, D\theta)\|_{C([0,1] \times [0,t_0])} \leq CM.$$

Then,

$$\begin{aligned} \int_0^t \int_0^1 |a_{1y}| (w_t^2 + w_y^2) dy ds &= \int_0^t \int_0^1 |\Theta_y a(U_y) + (\Theta + \bar{T}) a'(U_y) U_{yy}| (w_t^2 + w_y^2) dy ds \\ &\leq C(N) \int_0^t \int_0^1 (w_t^2 + w_y^2) dy ds \leq C(N) M^2 t. \end{aligned}$$

Every term on the right hand side of (3.20) involving $a_4, a_j, \partial a_j, j \neq 4$, can be handled in a similar way.

To estimate the other terms we use a slightly different technique. For exam-

ple, consider

$$\int_0^t \int_0^1 |a_{5tt} \theta_{tt}| |w_{yy}| dy ds,$$

where

$$a_{5tt} = \frac{2\kappa}{3} \Theta_{ytt} a^2 + 4\Theta_{yt} a a' U_{yt} + 2\Theta_y (a')^2 U_{yt}^2 + 2\Theta_y a a'' U_{yt}^2 + 2\Theta_y a a' U_{ytt}.$$

Since $(U, \Theta) \in X(N, t_0)$ and $(w, \theta) \in X(M, t_0)$, we have

$$\begin{aligned} & \int_0^t \int_0^1 |a_{5tt} \theta_{tt} w_{yy}| dy ds \\ & \leq C(N) \left(\int_0^t \int_0^1 |\theta_{tt} w_{yy}|^2 dy ds \right)^{1/2} \|D^3(U, \Theta)\|_{L^2([0,1] \times [0, t_0])} \\ & \leq C(N) N M \left(\int_0^t \int_0^1 w_{tt}^2 dy ds \right)^{1/2} \leq \sqrt{t} C(N) M N. \end{aligned}$$

Similarly, we can estimate the terms involving the second derivatives of a_j .

Then, it follows from (3.23) that

$$\begin{aligned} & \int_0^1 (|D^3 w|^2 + |D^2 \theta|^2) dy + \int_0^t \int_0^1 (|D^2 w_t|^2 + |D^2 \theta|^2 + |D^2 \theta_y|^2) dy ds \\ & \leq I(0) + M^2 C(N)(t + \sqrt{t}) \leq \frac{N^2}{2} + M^2 C(N)(t + \sqrt{t}). \end{aligned} \quad (3.24)$$

Now choose \tilde{t} sufficiently small so that $M^2 C(N)(t + \sqrt{t}) \leq \frac{N^2}{2}$. We then have $(w, \theta) \in X(N, \tilde{t})$.

Step 3. Now we show F is a contraction map in $X(N, \tilde{t})$ for sufficiently small $\tilde{t} \leq 1$.

Let $(U, \Theta), (\widehat{U}, \widehat{\Theta}) \in X(N, t_0)$. Then $(w, \theta) = F(U, \Theta)$, $(\widehat{w}, \widehat{\theta}) = F(\widehat{U}, \widehat{\Theta})$, and $(w - \widehat{w}, \theta - \widehat{\theta})$ satisfies

$$\begin{aligned} & (w - \widehat{w})_{tt} - a_1 (w - \widehat{w})_{yy} + a_3 (\theta - \widehat{\theta})_y + \frac{1}{\tau_p} (w - \widehat{w}) \\ & = (f - \widehat{f}) + (a_1 - \widehat{a}_1) \widehat{w}_{yy} - (a_3 - \widehat{a}_3) \widehat{\theta}_y, \\ & (\theta - \widehat{\theta})_t - a_2 (\theta - \widehat{\theta})_{yy} + a_4 (w - \widehat{w})_{yy} + a_5 (w - \widehat{w})_{yt} + a_6 (w - \widehat{w})_t + \frac{1}{\tau_w} (\theta - \widehat{\theta}) \\ & = (a_2 - \widehat{a}_2) \widehat{\theta}_{yy} - (a_4 - \widehat{a}_4) \widehat{w}_{yy} - (a_5 - \widehat{a}_5) \widehat{w}_{yt} - (a_6 - \widehat{a}_6) \widehat{w}_t. \end{aligned}$$

In a manner similar to deriving the energy estimates required for (3.24), and with the aid of Gronwall's inequality, we can obtain

$$\|(w - \widehat{w}, \theta - \widehat{\theta})\|_{X(N, \tilde{t})} \leq \tilde{t} C(N) \|(U - \widehat{U}, \Theta - \widehat{\Theta})\|_{X(N, \tilde{t})}.$$

Choose $\tilde{t} \leq 1$ sufficiently small such that $0 < \tilde{t}C(N) < 1$. Thus

$$F : X(N, \tilde{t}) \rightarrow X(N, \tilde{t})$$

is a contraction mapping on $X(N, \tilde{t})$ for sufficiently small $\tilde{t} > 0$.

The final step is to apply Banach's Contraction Mapping Theorem. We see that F has a unique fixed point in $X(N, \tilde{t})$. That is, (3.14)–(3.15) has a unique smooth solution (w, θ) on $0 \leq t \leq \tilde{t}$. Let $[0, t^*)$ denote the maximal interval of existence of this solution. If $t^* < \infty$ and (3.19) is not satisfied, then

$$\sup_{0 \leq t \leq t^*} \int_0^1 (|D^3 w|^2 + |D^2 \theta|^2) dy + \int_0^t \int_0^1 |\partial^2 \theta_y|^2 dy ds < \infty.$$

This inequality and use of the second equation in (3.16) imply that $(w, \theta)(y, t^*)$ satisfies the conditions for initial data in Theorem 2. Hence, the local existence can be continued to the interval $[0, t^* + \varepsilon)$ for some $\varepsilon > 0$. This contradicts the fact that $[0, t^*)$ is the maximal interval of existence. This completes the proof of Theorem 3.1. \square

3.2 A Priori Estimates via the Energy Method

In this section we use the energy method to make the *a priori* estimates for establishing the global existence of smooth solutions for the full hydrodynamic model (3.1)–(3.4). To achieve this, we derive the *a priori* estimates for the solution of (3.12)–(3.15), which is equivalent to (3.1)–(3.5).

Theorem 3 *Assume that*

$$\begin{aligned} 0 < \delta \leq \rho_0(x) \leq M, \quad (v_0, \theta_{0x})(i) = (0, 0), \quad i = 0, 1, \\ |2\tau_w - \tau_p| \leq M\sqrt{\tau_w \tau_p}. \end{aligned}$$

Then, given sufficiently small $\varepsilon > 0$, there exists $\varepsilon_0(\varepsilon) > 0$ such that, when

$$\|(\rho_0 - 1, v_0, d - 1)\|_{H^2(0,1)} + \|T_0 - 1\|_{H^3(0,1)} + \tau_p \leq \varepsilon_0,$$

we have, for any time interval $(0, t_0)$ on which the solution exists,

$$\begin{aligned} & \| (D^2 w, D^2 w_y, D\theta, D\theta_y, \tau_p w_{ttt}, \tau_p \theta_{tt}) \|_{L^2(0,1)}(t) + \| (\theta_y, \partial \theta_{yy}) \|_{L^2((0,1) \times (0, t_0))} \\ & + \frac{1}{\tau_p} \| (Dw_t, \partial w_{yy}, D\theta, D\theta_y) \|_{L^2((0,1) \times (0, t_0))} \\ & + \tau_p \| (\partial_y^2 w_y, w_{ttt}, D_y \theta_{tt}) \|_{L^2((0,1) \times (0, t_0))} \leq \varepsilon. \end{aligned} \quad (3.25)$$

Proof We start with the following equations from (3.12)–(3.13):

$$w_{tt} - a(w_y)^2(\theta + \overline{T})w_{yy} + a(w_y)\theta_y + \frac{w_t}{\tau_p} + w - \int_0^1 w dy = f, \quad (3.26)$$

$$\theta_t - \alpha a(w_y)\theta_{yy} + \alpha a(w_y)^2 \theta_y w_{yy} + \beta a(w_y)(\theta + \overline{T})w_{yt} + \frac{\theta}{\tau_w} + b \frac{w_t^2}{\sqrt{\tau_p \tau_w}} = 0, \quad (3.27)$$

with the initial-boundary conditions (3.14)–(3.15), where

$$f(y, t) = \int_0^1 \int_\eta^y (1 - \tilde{d})(1 + w_y) d\xi d\eta, \quad (3.28)$$

and

$$a(w_y) = \frac{1}{1 + w_y}, \quad b = -\frac{2\tau_w - \tau_p}{3\sqrt{\tau_p\tau_w}}, \quad \alpha = 2\kappa/3, \quad \beta = 2/3.$$

Then, from the assumptions of Theorem 3,

$$|b| \leq C.$$

We first assume that, for sufficiently small ε ,

$$\|(w, w_y, \theta)\|_{C^1([0,1] \times [0, t_0])} \leq \varepsilon \leq \frac{1}{2} \min(1, \bar{T}), \quad (3.29)$$

which implies

$$\bar{T}/2 \leq T \leq 3\bar{T}/2,$$

and

$$1/2 \leq w_y + 1 \leq 3/2 \quad \iff \quad 2/3 \leq \rho(x, t) \leq 2.$$

Then we prove that there exists an $\varepsilon_0 > 0$, independent of t_0 , such that, when

$$\|(\rho_0 - 1, v_0, d - 1)\|_{H^2(0,1)} + \|T_0 - 1\|_{H^3(0,1)} + \tau_p \leq \varepsilon_0,$$

the estimates (3.29) can be achieved. With this apriori argument, $a(w_y)$ is a C^∞ function of one variable w_y such that $|w_y| \leq \varepsilon < 1/2$ and $2/3 \leq a(w_y) \leq 2$.

We have from (3.26) and (3.14)–(3.15) that

$$((\theta + \bar{T})a^2 w_{yy} + f + \int_0^1 w dy)(i, t) = 0, \quad i = 0, 1. \quad (3.30)$$

We now derive the energy estimates. We first perform the following calculations: (3.26) \times ($\tau_p K^2 w$), (3.26) \times ($K^2 w_t$), (3.26) \times ($-\tau_p w_{yy}$), (3.26)_t \times ($K^2 w_{tt}$), (3.26)_y \times ($K^2 w_{yt}$), (3.26)_{tt} \times ($\tau_p^2 w_{ttt}$), (3.26)_{yt} \times w_{ytt} , (3.26)_{yy} \times ($\tau((\theta + \bar{T})a^2 w_{yy} + f + \int_0^1 w dy)$), (3.26)_{yy} \times ($K((\theta + \bar{T})a^2 w_{yy} + f + \int_0^1 w dy)_t$), (3.27) \times ($K^2 \theta$), (3.27) \times θ_t , (3.27)_y \times θ_y , (3.27)_{tt} \times ($\tau_w^2 \theta_{tt}$), (3.27)_{yt} \times θ_{yt} , and (3.27)_y \times θ_{yyy} ; then integrate over $[0, 1] \times [0, t]$, where $K = K(\gamma, k) \geq 1$ is determined later. Then we integrate by parts with the aid of the boundary conditions (3.15) and (3.30), followed by use of some elementary inequalities such as

$$ab \leq \delta a^2 + \frac{b^2}{4\delta}, \quad |w| = \left| \int_0^1 w_y dy \right| \leq \left(\int_0^1 w_y^2 dy \right)^{1/2}. \quad (3.31)$$

Finally, we add and re-group all the resulting inequalities from previous calcu-

lations and estimates together, and use $\tau_p \leq 1/2$ to obtain:

$$\begin{aligned}
& \int_0^1 [K^2|(D^2w, \theta)|^2 + K(((\theta + \bar{T})a^2w_{yy} + f)_y^2 + (w_{yy} + \frac{f + \int_0^1 wdy}{(\theta + \bar{T})a^2})_t^2) \\
& \quad + |D(w_{yt}, \theta, \theta_y)|^2 + \tau_p^2(w_{ttt}^2 + \theta_{tt}^2)](y, t) dy \\
& + \int_0^t \int_0^1 [K^2(\tau_p w_y^2 + \theta_y^2 + \frac{1}{\tau_p} |(Dw_t, w_{ytt}, \theta)|^2) + \frac{1}{\tau_p} (Kw_{yyt}^2 + |\partial(\theta, \theta_y)|^2) \\
& + \tau_p(w_{yy}^2 + \theta_{tt}^2 + |\partial w_{tt}|^2 + ((\theta + \bar{T})a^2w_{yy} + f)_y^2) + |\partial\theta_{yy}|^2 + \tau_p\theta_{ytt}^2] dy ds \\
& \leq C \int_0^1 \{K^2|(D^2w, \theta)|^2 + K[(\theta + \bar{T})a^2w_{yy} + f]_y^2 + (w_{yy} + \frac{f}{(\theta + \bar{T})a^2})_t^2 \\
& \quad + (|\theta_t| + |w_{yt}|)(|w_{yy}| + |f| + \int_0^1 |w| dy) \times \\
& \quad \times (|w_{yyt}| + |f_t| + \int_0^1 |w_t| dy + (|f| + \int_0^1 |w| dy)(|\theta_t| + |w_{yt}|)) \\
& + |D(w_{yt}, \theta, \theta_y)|^2 + \tau_w^2\theta_{tt}^2 + |D_t f|^2 + \tau_p^2w_{ttt}^2 + w_{yy}^2(\theta_t^2 + w_{yt}^2)\}(y, 0) dy \\
& + C \int_0^1 [\tau_p^2 K^2 w_t^2 + K w_{yt}^2 ((\theta + \bar{T})a^2 w_{yy} + f + \int_0^1 w dy)_y^2 + w^2 \\
& \quad + \tau_p^2 (w_{yt}^2 + w_{yyt}^2) + f_t^2 + w_{yt}^2 w_{yy}^2 + \tau_p |w_{yyt} f| \\
& \quad + \tau_p^2 (\theta_t^2 w_{yy}^2 + \theta_{yt}^2 w_{yy}^2 + \theta_y^2 w_{yyt}^2 + w_{yt}^2 \theta_y^2 w_{yy}^2)] dy \\
& + C \int_0^t \int_0^1 \{K^2 [\tau_p (|(D^2w_t, D^2\theta_y, \theta_{yt})|^2 + \tau_p |\partial_y f|^2 + (w^2 + w_y^2 + w_{yt}^2) w_{yy}^2) \\
& \quad + w_y^2 |w_{yt}| + w_t^4 + \tau_p w_{yy}^4 + \theta_y^2 |(w, Dw_y, \partial_y^3 w)|^2 \\
& \quad + (|\theta_t| + |w_{yt}|) w_{yt}^2 + |\theta| (|w_y w_{yt}| + |w_{yy} w_{yyt}|)] \\
& + K [\frac{1}{\tau_p} (w_t^2 + f_t^2 + \tau_p f_{tt}^2 + w_{yy}^2 (w_{yt}^2 + \theta_t^2) + (f + \int_0^1 w dy)^2 (w_{yt}^2 + \theta_t^2)) \\
& \quad + \tau_p w_{yt}^2 (f_t^2 + \int_0^1 w_t^2 dy) + |w_{yy}| (|f_t| + \int_0^1 |w_t| dy) \\
& \quad + (|w_{yt}| + |\theta_t|) (w_{yyt}^2 + w_{yt}^2 w_{yy}^2 + f_t^2 + \int_0^1 |w_t|^2 dy + w_{yt}^2 + \theta_t^2) \\
& \quad + |(\theta + \bar{T})a^2 w_{yy} + f + \int_0^1 w dy| (\tau_p (w_{ytt}^2 + w_{yt}^4 + \theta_{tt}^2 + \theta_t^2 w_{yt}^2) \\
& \quad + \frac{1}{\tau_p} (w_{yyt}^2 + w_{yt}^2 w_{yy}^2 + f_t^2 + w_t^2 + (f + \int_0^1 w dy)^2 (w_{yt}^2 + \theta_t^2))] \\
& + \tau_p w^2 + \tau_p w_{yt}^2 + |Dw_{yt}|^2 + \tau_p \theta_{yyy}^2 + \tau_p^2 w_{ttt}^2 + \tau_p^3 f_{tt}^2 \\
& + \tau_p^3 (w_{yy}^2 |Dw_{tt}|^2 + w_{yt}^2 (w_y^4 + w_{yy}^2 w_{yt}^2 + w_{ytt}^2)) + \tau_p w_{yyt}^2 |Dw_y|^2 \\
& + \tau_p^2 (w_{tt}^4 + w_t^2 w_{ttt}^2) + w_{yt}^2 (w_{yy}^2 + w_{tt}^2) + w_t^2 w_{ytt}^2
\end{aligned}$$

$$\begin{aligned}
& +\theta_y^2[\tau_p^3(w_{ytt}^2 + w_{yt}^4 + \theta_y^2 w_{ttt}^2) + w_{yt}^2 + w_{yyt}^2 + w_{yyy}^2 + w_{yy}^4 + w_{yt}^2 w_{yy}^2] \\
& +\theta_t^2[\tau_p^3(w_{yyt}^2 + w_{ytt}^2 + w_{yy}^2 w_{yt}^2 + w_{yt}^4) + w_{yt}^2] + \tau_p^3|\theta_t|w_{ytt}^2 \\
& +w_{yy}^2[\tau_p^3(\theta_{tt}^2 + \theta_{ytt}^2) + \tau_p(\theta_{yt}^2 + \theta_{yy}^2 + \theta_{yyt}^2 + \theta_{yyy}^2)] \\
& +w_{yt}^2[\tau_p^3(\theta_{yt}^2 + \theta_{yyt}^2 + w_{yt}^2 \theta_{yy}) + \theta_{yy}^2] + \tau_w^3 \theta_{yy}^2 w_{ytt}^2 \\
& +(|w_{yt}| + |\theta_t|)w_{yyt}^2 + \tau_w^2|\theta_{ttt}|(|\theta_y w_{yyt}| + |\theta_{yt} w_{yy}| + |w_{yt} w_{yy} \theta_y|) \\
& +|w_{yy} w_{yyt}|(|\theta| + \tau_p|\theta_t|) + |w_{yyt}|(|w_{yy}|w_{yt}^2 + |f_{tt}| + |w_{yy}\theta_{tt}|) \\
& +\tau_p^2|w_{yt}|w_{ytt}^2 + (|w_{yy}| + \tau_p|w_{yyt}|)(|f_t| + \int_0^1 |w_t|dy) \\
& +\frac{1}{\tau_p}(w_{yt}^2|Dw_y|^2 + w_t^2|(Dw_t, w_{yyt})|^2)\}dyds. \tag{3.32}
\end{aligned}$$

Using the elementary inequalities (3.31) and the *a priori* assumption (3.29), taking $K \geq K_0(C, \bar{T})$ and $\tau_p \leq \tau_0(C, \bar{T}, K)$ for sufficiently large $K_0 \geq 1$ and small $\tau_0 \leq 1/2$, and making tedious calculations, we have from (3.32):

$$\begin{aligned}
E_1(t) + \int_0^t G_1(s)ds & \leq CK^2(\|(\rho_0 - 1, v_0, d_1)\|_{H^2(0,1)}^2 + \|T_0 - 1\|_{H^3(0,1)}^2) \\
& +\varepsilon CK^2(E_1(t) + \int_0^t G_1(s)ds), \tag{3.33}
\end{aligned}$$

where

$$\begin{aligned}
E_1(t) & = \int_0^1 [K^2|(D^2w, \theta)|^2 + K|Dw_{yy}|^2 + |D(w_{yt}, \theta, \theta_y)|^2 + \tau_p^2(w_{ttt}^2 + \theta_{tt}^2)]dy, \\
G_1(t) & = \int_0^1 [K^2(\tau_p w_y^2 + \theta_y^2 + \frac{1}{\tau_p}|(Dw_t, w_{ytt}, \theta)|^2) + \tau_p(w_{yy}^2 + \theta_{tt}^2 + |(\partial_t w_{tt}, w_{yyy})|^2) \\
& +\frac{1}{\tau_p}(Kw_{yyt}^2 + |\partial(\theta, \theta_y)|^2) + |\partial\theta_{yy}|^2 + \tau_p\theta_{ytt}^2]dyds. \tag{3.34}
\end{aligned}$$

In the process of these calculations, we used the following identities:

$$\tilde{d}_t = -w_t d_x, \quad \tilde{d}_y = d_x / \rho, \quad \tilde{d}_{tt} = -w_t^2 d_{xx} - w_{tt} d_x,$$

and the following inequalities:

$$\begin{aligned}
|f_y| & \leq |1 - \tilde{d}| + |(\tilde{d} - 1)w_y|, & |f_{yt}| & \leq |w_{yt}(\tilde{d} - 1)| + |(w_y + 1)w_t d_x|, \\
|f_t| & \leq \int_0^1 |f_{yt}|dy + \int_0^1 |w_{yt}|dy \int_0^1 |f_y|dy + \int_0^1 |w_y + 1|dy \int_0^1 |f_{yt}|dy, \\
|f_{tt}| & \leq (2 + \int_0^1 |w_y|dy) \int_0^1 (|w_y + 1|(w_t^2 |\tilde{d}_{xx}| + |w_{tt} \tilde{d}_x|) + 2|w_{yt} w_t \tilde{d}_x| \\
& +|\tilde{d} - 1||w_{ytt}|)d\eta + \int_0^1 |w_{ytt}| \int_0^1 |f_y|dy + 2 \int_0^1 |w_{yt}| \int_0^1 |f_{yt}|dy.
\end{aligned}$$

Then, if ϵ is sufficiently small, we have

$$E(t) + \int_0^t G(s) ds \leq C(\|(\rho_0 - 1, v_0, d - 1)\|_{H^2(0,1)}^2 + \|T_0 - 1\|_{H^3(0,1)}^2), \quad (3.35)$$

where

$$\begin{aligned} E(t) &= \int_0^1 (|D^2(w, w_y)|^2 + |D(\theta, \theta_y)|^2 + \tau_p^2(w_{ttt}^2 + \theta_{tt}^2)) dy, \\ G(t) &= \int_0^1 \left(\frac{1}{\tau_p} (|(Dw_t, \partial w_{yy})|^2 + |D(\theta, \theta_y)|^2) \right. \\ &\quad \left. + \tau_p (|\partial_y^2 w_y, w_{ttt}|^2 + |(\theta_y, \partial \theta_{yy})| + \tau_p |\partial_y \theta_{tt}|^2) \right) dy. \end{aligned} \quad (3.36)$$

By the Sobolev inequality we obtain

$$\|D(Dw, \theta)\|_{C([0,1] \times [0, t_0])} \leq \Gamma \sup_{0 \leq t \leq t_0} \sqrt{E(t)},$$

where Γ is a positive constant. Hence if

$$\|(\rho_0 - 1, v_0, d_0 - 1)\|_{H^2(0,1)} + \|\theta_0\|_{H^3(0,1)} \leq \frac{\epsilon}{\Gamma \sqrt{C}}, \quad (3.37)$$

we have

$$\|D(Dw, \theta)\|_{C([0,1] \times [0, t_0])} \leq \Gamma \sqrt{C} (\|(\rho_0 - 1, v_0, d_0 - 1)\|_{H^2(0,1)} + \|\theta_0\|_{H^3(0,1)}) \leq \epsilon$$

and the assumption for the function $a(w_y)$ is satisfied. This completes the proof of Theorem 3. \square

3.3 Global Existence, Uniqueness, and Asymptotic Decay

Theorem 4 *Assume that*

$$0 < \delta \leq \rho_0(x) \leq M, \quad (v_0, \theta_{0x})(i) = (0, 0), \quad i = 0, 1, \quad |2\tau_w - \tau_p| \leq M \sqrt{\tau_w \tau_p}.$$

Also, assume that the following norms,

$$\|(\rho_0 - 1, v_0, d - 1)\|_{H^2(0,1)}, \quad \|\theta_0\|_{H^3(0,1)},$$

and both relaxation times are sufficiently small. Then, (3.1)–(3.4) has a unique global smooth solution $(\rho, v, T, \phi)(x, t)$ satisfying

$$\|(\rho - 1, \rho_x, \rho_t, v, v_x, v_t, T - \bar{T}, T_x, T_t)\|_{C[0,1]}(t) \rightarrow 0 \quad \text{when } t \rightarrow \infty.$$

Proof We now use the continuation argument to prove the global existence. From the local existence and uniqueness theorem, there exists an interval $[0, t_0)$ such that (3.1)–(3.4) has a unique smooth solution. Then, Theorem 3 indicates that

$$E(t_0) \leq \epsilon/\Gamma.$$

At this point, we can apply the local existence and uniqueness theorem in previous sections to continue the solution to an interval $[t_0, t_1]$, $t_0 < t_1$. We can continue this process onto $[0, \infty)$ unless the sequence $\{t_n\}$ converges to a finite number $t^* < \infty$ as $n \rightarrow \infty$. For such a t^* we must have a maximal interval of existence $[0, t^*)$; i.e., (3.19) holds. This contradicts *a priori* estimate (3.35). Thus there can be no such $t^* < \infty$ and

$$\|(w_y, \theta)\|_{C([0,1] \times [0, \infty))} \leq \varepsilon$$

holds. Then we have shown the existence of unique global smooth solution satisfying

$$\begin{aligned} (D^3 w, D^2 \theta) &\in L^2([0, \infty); L^2(0, 1)); \\ (D^2 w, D\theta)(y, t) &\rightarrow 0 \quad \text{uniformly as } t \rightarrow \infty. \end{aligned}$$

By (3.11), we obtain

$$\|(\rho - 1, \rho_x, \rho_t, v, v_x, v_t, T - \bar{T}, T_x, T_t)\|_{C[0,1]}(t) \rightarrow 0 \quad \text{when } t \rightarrow \infty.$$

This completes the proof of Theorem 4. \square

4 Singular Relaxation Limit to Drift-Diffusion Equations

In this section we study the singular limit of (3.1)–(3.4) when the relaxation times τ_w and τ_p tend to zero with $0 \leq 2\tau_w - \tau_p \leq M\sqrt{\tau_p\tau_w}$. We first scale the variables (ρ, v, T, ϕ) , and then show that the limit functions satisfy the drift-diffusion system as $\tau_p, \tau_w \rightarrow 0$.

Let

$$\begin{aligned} \rho^\tau(x, s) &= \rho\left(x, \frac{s}{\tau_p}\right), \quad v^\tau(x, s) = \frac{1}{\tau_p}v\left(x, \frac{s}{\tau_p}\right), \\ \theta^\tau(x, s) &= \frac{1}{\tau_w}\left(T\left(x, \frac{s}{\tau_w}\right) - \bar{T}\right), \\ \phi^\tau(x, s) &= \phi\left(x, \frac{s}{\tau_p}\right), \quad d^\tau(x, s) = d\left(x, \frac{s}{\tau_p}\right). \end{aligned}$$

Then (3.1)–(3.4) is transformed as follows.

$$\rho_s^\tau + (\rho^\tau v^\tau)_x = 0, \tag{4.1}$$

$$\tau_p^2 v_s^\tau + \tau_p^2 v^\tau v_x^\tau + \frac{1}{\rho^\tau}((\tau_w \theta^\tau + \bar{T})\rho^\tau)_x + v^\tau = \phi_x^\tau, \tag{4.2}$$

$$\tau_w^2 \theta_s^\tau - \frac{2\kappa\tau_w}{3\rho^\tau} \theta_{xx}^\tau + \tau_p \tau_w v^\tau \theta_x^\tau + \frac{2\tau_p}{3}(\tau_w \theta^\tau + \bar{T})v_x^\tau - \frac{\tau_p(2\tau_w - \tau_p)}{3\tau_w}(v^\tau)^2 + \theta^\tau = 0, \tag{4.3}$$

$$\phi_{xx}^\tau = \rho^\tau - d^\tau, \tag{4.4}$$

with the following initial-boundary data:

$$\begin{aligned}(\rho^\tau, v^\tau, \theta^\tau) &= (\rho_0(x), v_0(x), T_0(x) - \bar{T}), \\(v^\tau, \theta_x^\tau, \phi^\tau)(i, s) &= 0, \quad i = 0, 1, \quad s \geq 0.\end{aligned}$$

We assume that initial data $(\rho_0(x), v_0(x), T_0(x) - \bar{T})$ are independent of τ_p and τ_w .

Theorem 5 *Assume that the conditions of Theorem 2 hold. Let (ρ, v, T, ϕ) be the solution of (3.1)–(3.4) constructed in Theorem 4. Then there exists $N(x, s)$ such that*

$$(\rho^\tau, \phi_x^\tau) \rightarrow (N, \int_0^1 \int_\eta^x (N(\xi, s) - d(\xi)) d\xi d\eta), \quad \text{a.e. as } \tau_p, \tau_w \rightarrow 0.$$

The limit function N satisfies the drift-diffusion equation,

$$N_s + \left(N \int_0^1 \int_\eta^x (N(\xi, s) - d(\xi)) d\xi d\eta - N_x \right)_x = 0,$$

in the sense of distributions.

Proof To show the convergence of subsequence $\{\rho^\tau, v^\tau, \theta^\tau\}$, we need some estimates which are independent of τ_p and τ_w .

Notice that the scaling sequence $(\rho^\tau, v^\tau, \theta^\tau, \phi^\tau)$ satisfies (4.1)–(4.4). Then the relations of the scaling sequence and the function (w, θ) , satisfying (3.26)–(3.27), are as follows, where we use τ for both τ_p and τ_w :

$$\begin{aligned}\rho^\tau(x, s) &= \frac{1}{w_y(y(x, \frac{s}{\tau}), \frac{s}{\tau}) + 1}, & v^\tau(x, s) &= \frac{1}{\tau} w_t \left(y \left(x, \frac{s}{\tau} \right), \frac{s}{\tau} \right), \\ \theta^\tau(x, s) &= \frac{1}{\tau} \theta \left(y \left(x, \frac{s}{\tau} \right), \frac{s}{\tau} \right), \\ \rho_x^\tau(x, s) &= -(\rho^\tau)^3 w_{yy} \left(y \left(x, \frac{s}{\tau} \right), \frac{s}{\tau} \right), & v_x^\tau(x, s) &= \frac{1}{\tau} \rho^\tau w_{yt} \left(y \left(x, \frac{s}{\tau} \right), \frac{s}{\tau} \right), \\ \rho_{xx}^\tau(x, s) &= -(\rho^\tau)^4 w_{yyy} \left(y \left(x, \frac{s}{\tau} \right), \frac{s}{\tau} \right) + 3(\rho^\tau)^5 \left(w_{yy} \left(y \left(x, \frac{s}{\tau} \right), \frac{s}{\tau} \right) \right)^2, \\ \phi^\tau(x, s)_x &= \int_0^1 (w_y + 1) \left(\int_\eta^{y(x, \frac{s}{\tau})} (1 - (w_y + 1) \tilde{d}) \left(\xi, \frac{s}{\tau} \right) d\xi \right) d\eta, \\ \phi^\tau(x, s)_{xx} &= \rho^\tau(x, s) - d(x), \\ \phi^\tau(x, s)_{xs} &= -(\rho^\tau - d) v^\tau(x, s) + \int_0^1 \frac{w_{yt}}{\tau} \left(\int_\eta^y (1 - (w_y + 1) \tilde{d}) \left(\eta, \frac{s}{\tau} \right) d\eta \right) dy \\ &\quad - \int_0^1 (w_y + 1) \left(\int_\eta^y \frac{1}{\tau} (w_{yt} \tilde{d} - \tilde{d}_x (w_y + 1) w_t) \left(\xi, \frac{s}{\tau} \right) d\xi \right) d\eta.\end{aligned}$$

Therefore,

$$\|(\rho^\tau, \phi^\tau, \phi_x^\tau)\|_{L^\infty} + \int_0^\infty \int_0^1 [(v^\tau)^2 + (\theta^\tau)^2 + (\rho_s^\tau)^2 + (\rho_x^\tau)^2 + (\phi_{xx}^\tau)^2 + (\phi_{xs}^\tau)^2] dy ds \leq C,$$

where $C > 0$ is independent of τ .

Thus, there exists $N(x, s)$ such that

$$(\rho^\tau, \phi_x^\tau) \rightarrow (N, \Phi_x) \quad \text{a.e. as } \tau \rightarrow 0,$$

where $\Phi_x(x, s) = \int_0^1 \int_\eta^x (N(\xi, s) - d(\xi)) d\xi d\eta$.

From (3.1)–(3.2),

$$\begin{aligned} & \int \int (\rho^\tau \psi_s + \rho^\tau v^\tau \psi_x) dx ds = 0, \\ & \tau_p \int \int (\rho^\tau v^\tau \psi_{xs} + \tau_p \rho^\tau (v^\tau)^2 \psi_{xx}) dx ds + \int \int \rho^\tau (\tau_w \theta^\tau + \bar{T}) \psi_{xx} dx ds \\ & = \int \int (\rho^\tau v^\tau - \rho^\tau \phi_x^\tau) \psi_x dx ds, \end{aligned}$$

for all $\psi \in C_0^\infty(R \times R_+)$. That is, for any test function $\psi(x, t) \in C_0^\infty((0, 1) \times (0, \infty))$, the relation,

$$\begin{aligned} & \int \int (\rho^\tau \psi_s + \rho^\tau \phi_x^\tau \psi_x + \bar{T} \rho^\tau \psi_{xx}) dx ds \\ & = - \int \int (\tau_p^2 \rho^\tau v^\tau \psi_{xs} + (\tau_p \rho^\tau (v^\tau)^2 + \tau_w \rho^\tau \theta^\tau) \psi_{xx}) dx ds, \end{aligned}$$

holds. Let $\tau_p, \tau_w \rightarrow 0$. Since

$$\|\rho^\tau\|_{L^\infty((0,1) \times (0,\infty))} + \|(v^\tau, \theta^\tau)\|_{L^2((0,1) \times (0,\infty))} \leq C,$$

where C is independent of τ_p, τ_w , and t , then

$$\int \int (N \psi_s + N \Phi_x \psi_x + \bar{T} N \psi_{xx}) dx ds = 0.$$

This completes the proof of Theorem 5. □

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