Some Thoughts on View Camera Calculations

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The purpose of this article is to explore how simple geometric optics may help a large format photographer. It is aimed at readers who are comfortable with formulas and simple algebra and are familiar with the basics of how to use a view camera, but it doesn't require anything beyond that. I wrote it to clarify the basic principles in my own mind, and I hope it may be useful for some others. I explore how certain simple formulas can enhance one's technique, but of course one must always keep in mind the primacy of the image on the ground glass and what it says. This is also a work in progress, and it may very well contain mistakes or misconceptions. If you find anything like that, please report it to me. I was inspired in part by Bob Wheeler's notes, and the reader who wants to learn more should look there. (www.bobwheeler.com/photo). Very little of this is original with me, and that part may be wrong, but I've found that some of it doesn't seem universally known.

1 Measuring along the rail

I used a Horseman Technical Camera (6 x 7 format) for many years before finally getting a Toho 4 x 5 view camera in 2002. The Horseman has limited front and back movements, so I had mastered the basics of view camera technique. But I found there were some significant differences between what worked for the Horseman and what I now do. As a professional mathematician, I am fascinated by optical formulas. But, even for me, formulas have to be very simple to be useful in the field. One of my aims was to derive a set of methods which could be applied using mental arithmetic, or at worst the simplest possible calculator. That is not always possible, but it is surprising how often it is.

The first thing I discovered was how useful it is to have a distance scale on the rail of a large format camera. The Horseman has no such scale and I never missed it. The Toho also lacked a scale, but at the suggestion of Peter Desmidt, I got a metric adhesive backed metal tape at a local woodworking tool shop, which I taped to the rail after removing one of the Toho logos. (See Figure 1.) (Such scales can also be purchased through Amazon.com, and they may be cut up for several cameras or several photographers.) Initially, I just used the scale to help set the distance between the standards when setting up the camera or changing lenses. But then I began to think about other ways I could use it. One problem was apparent from the start. Often one is interested in displacements along the rail as one

Figure 1: Adding a metric scale to the rail

moves the front or rear standard. These displacements may be only a few millimeters, and it is hard to estimate them accurately. The way I solved that problem for my Toho was to put a scale on the focusing knob. (Some view cameras may already have such a scale, but the Toho didn't.) Making some measurements, I found that one rotation of the focusing knob produces a 20 mm displacement along the rail. The circumference of the focusing knob is just about 95.5 mm. So I cut a narrow piece of artist's tape that long, put 20 equally spaced marks on it, and taped it to the focusing knob. I put another small piece of tape with a reference mark just under the focusing knob, and I was ready to go. Initially, I didn't attempt to do this with extreme precision, but the almost five times expansion in scale made it possible to estimate distances along the rail to better than half a millimeter. Later I realized I could make a more accurate scale by photographing 20 cm of a metric ruler, rescaling the image in a photoeditor and printing it. (See Figure 2.) The major problem in using such a scale is lining it up with the reference mark, but with good eyes one should be able to estimate distances to a few tenths of a millimeter, about comparable to the typical focusing error I make when using a loupe. The Toho focusing knob also has a small amount of play when tightened down, so I always turn it to the right before reading the scale. Let me describe some of the things it is possible to do with such an arrangement. Since the Toho focuses by moving the rear standard, I will assume that is what you are doing, but it works pretty much the same if you focus by moving the lens.

2 Using the camera as a rangefinder

I will start off with the assumption that the film plane, the lens plane, and the subject plane are all parallel. If the lens plane is tilted or swung, it gets a bit more complicated although some of the same principles apply. I will come back to that later. I will also assume that the film lies flat in a plane and the film plane can be placed exactly where it should be. Moreover, I will assume that exact geometric optics applies, which means that lens aberrations and diffraction effects will be ignored. Later I will say a bit about how diffraction might affect the results, and how focusing errors might affect the calculations.

First, one can use the camera as a simple rangefinder. Let f be the lens focal length.

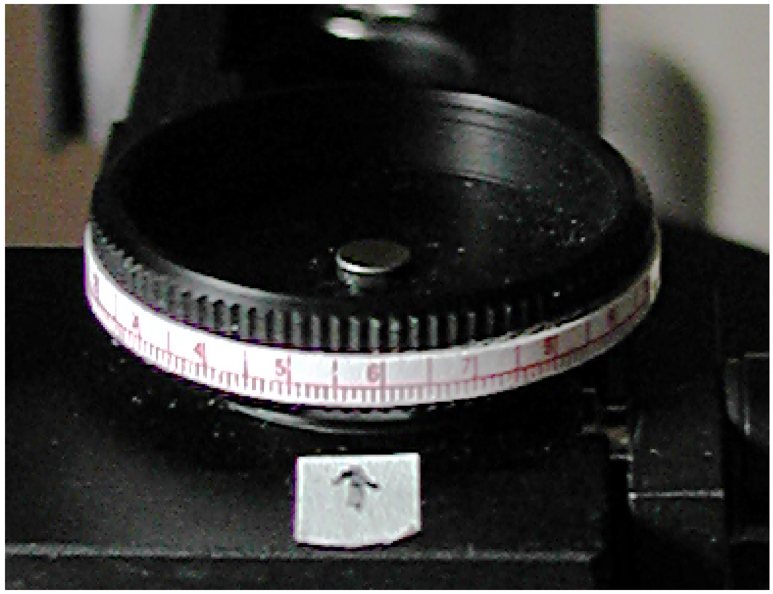

Figure 2: Scale on the focusing knob

Assume you are focused on a subject plane at a certain distance u from the lens plane. Let d be the extension beyond f, i.e., beyond the position when the lens is focused at infinity. Then for sufficiently distant objects, the distance to the subject plane is given quite accurately by

$$u = \frac{f^2}{d}.$$

For example, suppose one is using a 90 mm lens, and the extension beyond infinity focus is 2 mm. Then, the subject plane must be just about at $\frac{8100}{2} = 4050$ mm $= 4.05$ meters.

If one is doing closeup work, then the exact formula is

$$u = \frac{f(f+d)}{d},$$

which is slightly harder to compute with.

Cameras and lenses for 35 mm and medium format cameras usually have distance scales which tell you how far the subject plane is. One could improvise such scales for a view camera, one for each lens, so the subject distance could be read directly without calculating anything. But it has become increasingly apparent to me that the distance to the subject plane is often not relevant. All the information you need is available through measurements made on the camera directly. So the linear distance scale on the rail, as expanded on the focusing knob, is all you really need.

3 Hyperfocal technique

Let me illustrate that principle by discussing the hyperfocal distance. As most photographers know, if you focus at the hyperfocal distance, then everything from half that distance to infinity will, in principle, be in focus. What does the hyperfocal distance correspond to as a displacement along the rail? That requires a little algebra. The usual formula for the hyperfocal distance is

$$h = \frac{f^2}{Nc}$$

where f, as above, is the focal length, N is the f-number, and c is the diameter of the largest allowable circle of confusion in the film plane. The circle of confusion bounds a disc in the film plane which results when the exact image point is a slight distance from the film plane. If that disc is small enough, after enlargement in a final print, it won't be distinguishable from a point. The proper value of c depends on the format, and there is considerable difference of opinion about what it should be for any given format. For 4 x 5 photography, the acceptable range seems to be 0.05 mm to 0.125 mm. In this article I will generally use $c = 0.1$ mm, but there is nothing magic about that, and you may prefer another value. (See 3.1.)

Assume now that you are focused at the hyperfocal distance so that $u = h$. Compare the two formulas

$$h = \frac{f^2}{d} \text{ and } h = \frac{f^2}{Nc}.$$

Since the numerators are the same, the denominators must also be the same, i.e.,

$$d = Nc. \tag{1}$$

In other words, to focus at the hyperfocal distance, set the rear standard N times c units beyond the position for focusing at infinity.

What about half the hyperfocal distance? In that case, $u = \frac{h}{2}$. If the extension along the rail beyond infinity for the half the hyperfocal distance is d', we have

$$\frac{h}{2} = \frac{f^2}{d'}$$

so it follows by some simple algebra, that $d' = 2d = 2Nc$. See Figure 3 for a picture of this. You see that when the rear standard is displaced $d = Nc$ units from the infinity focus position, i.e., the lens to film distance is $v = f + d$, then any image point which lies within d units of either side of the film plane will produce a sufficiently small blur in the film plane.

Note that you can now forget about the hyperfocal distance. The distance $d = Nc$ is independent of focal length, and it is really the only number you need to know in order to apply hyperfocal technique. Of course, if you want, you can double check by the usual methods, provided you have some method to measure the distance to the subject plane precisely. Let's look at an example. Suppose, like me, you take $c = 0.1$, and you stop down to f/22. Then you should set the rear standard 22 (0.1) = 2.2 mm beyond the infinity setting to be focused on the hyperfocal distance for any focal length lens. For a 90 mm lens, that corresponds to a subject plane at $\frac{8100}{2.2} = 3.68$ meters. Of course, it is hard to position the

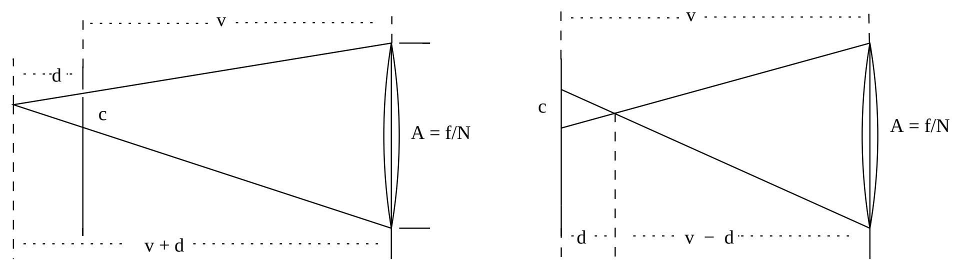

Figure 3: The basic diagram

rear standard at precisely 2.2 mm beyond the infinity position, even with an aid such as my expanded scale. So you will certainly want to examine the image with a loupe and check the focus before stopping down, and then check the range of adequate focus after stopping down. If you have a laser rangefinder or you take the trouble to measure with a tape, you can try focusing precisely at the hyperfocal distance, in this case 3.68 meters. But it is not really much easier to do that, since, even with a loupe, it is hard to focus to better than a tolerance of a few tenths of a mm along the rail. (See Section 8 below on focusing errors.) In either case, you are limited by how precisely you can position the standard.

3.1 The choice of circle of confusion

As noted above, I chose $c = 0.1$ mm. The basis for this choice is that someone with normal vision, looking at a print at 10 to 12 inches, can resolve about 5 lp/mm. This corresponds to a circle of confusion in the print of diameter $\frac{1}{5} = 0.2$ mm. If I take as my standard an 8 x 10 print viewed at that distance, then the 4 x 5 film image has to be enlarged about twice, which then requires for the film $c = \frac{0.2}{2} = 0.1$ mm. The same standard will apply for larger prints if they are viewed from proportionately further away. If one believes the proper distance for viewing a print is the diagonal of the print, then that makes sense, and taking $c = 0.1$ mm for a 4 x 5 format should suffice. Of course, for larger formats, such at 5 x 7 or 8 x 10 film, proportionately larger values of c would be appropirate.

On the other hand, many of us are inevitably drawn in to prints and find ourselves scanning for detail from very close up. If you expect that to happen, a circle of confusion of diameter 0.1 mm will not be satisfactory. Indeed, it is not likely that any value of c will be entirely satisfactory in all circumstances. For example, taking $c = 0.05$ will allow viewers to examine a 16 x 20 inch print closely but not a 32 x 40 inch print. Also, diffraction will certainly be a factor for large prints viewed very closely. The usual rule of thumb is that a perfect lens will resolve about $\frac{1500}{N}$ lp/mm. At f/32, that is about 47 lp/mm , and in an eight times enlargement, that would be reduced to a little under 6 lp/mm. And this ignores other factors such as film resolution.

4 Getting what you want to be sharp in focus

The amazing thing is that the same principle mostly works at any distance, not only at the hyperfocal distance. (But see below for exact formulas and how in certain cases, such as closeups, one must modify the method.) In more detail, suppose the rear standard is positioned for exact focus on some subject plane at another distance v from the lens. Then, any image point which is within the distance $d = Nc$ on either side of the film plane will project a disc in the film plane which is not distinguishable from a point. That means the corresponding subject point will be within the acceptable depth of field. If you want, you can calculate the depth of field, but it is not necessary to do so.

That leads to an old method for focusing a view camera. Focus on the furthest point you want to be in focus, then on the nearest point, and note the two positions on the scale. Then, place the standard halfway in between. This has just exactly the desired effect; namely the distance of the film plane from either position is the same distance and the film plane is symmetrically placed between. As above, that distance is $d = Nc$.

It goes without saying that you can't follow this rule blindly. Your estimate of the positions of the near and far focus points be will be off slightly due to focusing error. You can decrease this by using a loupe, but you can't eliminate it. (See Section 8 on focusing errors for further discussion of this issue and possible ways to minimize it.) If the distance between the near and far focus points is small, the focusing error will eat up a considerable part of the range. Also, other factors may affect where you want to focus. So you certainly have to check this by what you see on the ground glass.

We can use these facts to determine what the f-stop should be to keep everything in focus, at least for relatively large apertures where diffraction is not much of a problem. Let w be the distance along the scale between the positions for near and far focus, or what is usually called the *focus spread*. As just noted, $d = Nc = \frac{w}{2}$, so

$$N = \frac{w}{2c}.$$

Let's try an example. Suppose you find that the focus spread is w = 3 mm, and as above you use $c = 0.1$. Then N should be $\frac{3}{2(0.1)} = 15$. Thus if you stop down to f/16, that should suffice. Again, in practice you would check this on the ground glass at the taking aperture and perhaps make some adjustments. The calculation is based on the assumptions that you have a perfect lens obeying the strict laws of geometric optics, and that you can focus perfectly at the near and far focus points. Real lenses have aberrations and there is always some focusing error, so you may want to stop down one or two additional stops to be sure, to the extent that subject motion and diffraction effects allow.

The arithmetic is not so easy if you don't choose a simple value of c like 0.1mm. In that case, it is easy enough to prepare a table which for different values of the focus spread tells you the f-number to use.

If the focus spread is large, i.e., you are aiming for a lot of depth of field, then, as the formula indicates, you will have to stop down considerably. If so, diffraction will become a significant factor, and you may want to use another approach. (See below.) In addition, it may be difficult to judge visually what is in focus because the image will be so dim. Using a loupe can help, but it should be remembered that doing so may decrease the apparent depth of field because with high magnification, you are in effect choosing a very small circle of confusion, perhaps smaller than you intended. For example, looking at at a 4 x 5 image with a 4 X loupe corresponds roughly to viewing a 16 x 20 inch print from about 10 to 12 inches, and looking at it with an 8 X loupe corresponding to viewing a 32 x 40 inch print at that distance.

It should also be noted that some people don't believe that dividing the focus spread equally produces the best results. There seems to a lot of debate about it, and reasonable arguments can be made for other ways to do it. For example, it might be argued that you should be more critical about sharpness for small distant objects than for large nearby objects. That means you should want a smaller circle of confusion for image points closer to the lens than for image points further from the lens. That of course changes the mathematics. If you want the largest circle of confusion for distant points to be half that for near points, then the proper subdivision of the focus spread changes. The focus should be set at $\frac{w}{3}$ from the distant focus point and $\frac{2w}{3}$ from the near focus point. (See Joe Englander.) Others would see this as an unacceptable waste of some of the usable depth of field. (However, there is a plausible argument for always favoring the distant focus slightly, particularly for extreme wide angle lenses. See Section 7.)

How does all this apply to close up objects? If the focus spread between the near and far focus is not too large, then all you need to do is to replace the f-number N by the effective f-number

$$N_e = N(1 + M)$$

where M is the scale of reproduction. This is the same quantity that is used to make exposure adjustments for closeups. You can still use the formula

$$d = N_e c$$

as above, and nothing else changes. The actual f-number is given by

$$N = \frac{N_e}{1 + M} = \frac{w}{2c}\frac{1}{1 + M}.$$

If you ignore this factor $1 + M$, you overestimate the needed f-number, which in many situations is innocuous.

It should be noted that you can determine the factor $1 + M$ from measurements along the rail. Namely, if d_∞ is the extension of the lens from infinity when focused on the subject, then

$$1 + M = 1 + \frac{d_\infty}{f}.$$

You may also estimate M by putting a circular disk of known diameter at the plane of principal focus and measuring its longest dimension on the ground glass.

(See www.salzgeber.at/disc for an application of this principle for estimating exposure correction.)

However, when working with extreme wide angle lenses and small apertures, the focus spread w may be a large fraction of the focal length, particularly for closeups. In that case the approximate formulas I've been using are no longer accurate enough, and you have to use exact formulas instead. The analysis is more complex, and not so easy to apply with simple arithmetic. I discuss this in Section 7.

4.1 Brief remarks on diffraction

In our discussion so far, we have ignored diffraction. Even a perfect lens is limited by diffraction and if you stop down enough, the image of a point—the so called Airy disc—may be comparable in size to whatever circle of confusion you set for yourself. The smaller you set the circle of confusion, the sooner that will happen. The overall effect of diffraction is to degrade resolution throughout the image, so its effect on the perception of what is sharp in the image can be complicated. It appears that large format photographers differ in how they approach diffraction and how seriously they take it. Many feel that you should not let the specter of diffraction dissuade from using very small apertures if you need them to get adequate depth of field.

In our discussions, we have specified a largest acceptable circle of confusion and used the formula $N_e = \frac{w}{2c}$ to determine the f-number. This method may be modified to account for diffraction in a variety of ways. A complete analysis requires the use of modulation transfer functions which give the complete spatial frequency response for each factor. But there are two simple rules of thumb which are often used instead to estimate the combined effect. You can just add the diameters of the discs or you can take the square root of the sum of the squares. Using either one, you pick N so the combined effect is not greater than your acceptable error. See the link to Peterson at www.largeformatphotography.info/fstop.html for a discussion of how to do that based on the second rule.

That web site also discusses an alternate approach, based on an article of Paul Hansma. Instead of fixing a maximal allowable error disc, the f-stop is chosen so that the combined effect of diffraction and geometric defocus at the limits of the depth of field is as small as possible. It turns out that either rule of thumb for combining errors yields the same formula for the optimal f-number

$$N_e = \sqrt{375w}$$

where, as above, w is the focus spread. For small focus spreads, this will tend to give a larger f-number than may be necessary. The aperture will be relatively large in such cases, and diffraction is neglible for typical degrees of enlargement. So the formula $N_e = \frac{w}{2c}$ gives a reasonable estimate, particularly if you stop down an additional stop. (Of course, it won't hurt to stop down more, if you can, since that will allow making very large prints which can be viewed very closely.) On the other hand this method may give a more realistic f-number for a large focus spread, since at smaller apertures diffraction plays a more prominent role.

Wheeler, using half the estimate for the contribution from the Airy disc, comes up instead with

$$N_e = \sqrt{750w}.$$

Wheeler bases his estimate on the Rayleigh criterion for when two points will be resolved, while Hansma uses the diameter of the Airy disc.

Both Hansma and Wheeler cite observational verification for their estimates. I don't claim to understand the matter very well, but it seems to me that any simple scheme for combining the Airy disc with the circle of confusion from defocus is questionable. But these analyses do show that for large focus spreads which would entail relatively small apertures, diffraction will limit resolution enough that very large prints viewed very close up are bound to show some fuzziness.

Note that, as before, the formulas give the necessary effective f-number, and the actual f-number would be obtained by dividing that by $1 + M$.

4.2 Setting the f-stop precisely

F-stops on lenses are usually marked in 'fractions' of an f-stop. The relation between the ratio of f-number N to N_0 and the corresponding fraction of an f-stop is

$$\nu = \frac{2 \log(N/N_0)}{\log 2}.$$

It doesn't matter which base logarithm you use. For example, if the f-number is 40, that corresponds to $\frac{2\log(40/45)}{\log 2} = -.34$ f-stops. In other words, it is just about one third stop less than f/45.

For convenience, here is a table of f-numbers and the corresponding displacement, up or down, from the nearest marked f-stop.

f-number	stop
5	5.6-.33
10	11-.28
15	16-.19
20	20-.28
25	22+.37
30	32-.19
35	32+.26
40	45-.34
45	45+.00
50	45+.30
55	64-.44
60	64-.09

Note that the the two numbers in the right column are in different scales. The first is linear, while the second is logarithmic. Some digital exposure meters will show the f-number 35 as 32.26, but it seems to me this is bound to be confusing if the user doesn't understand just what the numbers mean.

Here is a table giving Hansma's estimates for f-number and f-stop for different focus spreads.

w	f-number	stop
1	19.4	22-.36
1.5	23.7	22+.22
2	27.4	32-.22
3	33.5	32+.14
4	38.7	45-.45
5	43.3	45-.11
6	47.4	45+.15
7	51.2	45+.38
8	54.8	64-.43
9	58.0	64-.26
10	61.2	64-.13
11	64.2	64+.00

5 Tilted lens plane

When the lens plane is parallel to the film plane, the plane of principal focus is parallel to both of them, and you have to rely on depth of field to get everything you want in focus. By changing the angle that the lens plane makes with the film plane you can alter the position of the subject plane so as to improve sharpness for important parts of the subject. This may be done by tilting or/and swinging either the lens or the back, but for convenience I will assume the lens is being tilted. The Scheimpflug rule tells us that the film plane, the lens plane, and the plane of exact focus in the subject all intersect in a line. Visualizing the location of that line is the first step, but determining the tilt angle can be tricky.

There are broadly two extreme cases that one encounters, and I will consider them apart, although in practice it may sometimes be hard to separate them.

5.1 Primary subjects in a plane

It often happens that the primary subjects of interest lie very close to a single plane. In this case, if we get the tilt angle right, then we need not stop down very far in order to get everything we want in focus.

Wheeler has derived a simple method for determining how far to tilt the lens if you know where you want to place subject plane. I've known this method for a while, but I never used it, because making the measurements with my Horseman was not easy. But I recently tried it with my modified Toho, and I was surprised how well it works.

It has several forms, but the one I find most useful is the following. It works if substantially all the subject points are distant from the lens. With the lens plane and film plane parallel, find two points, near and far, which you want to be in focus in the desired subject plane. Find the distance between them along the rail; call that W. Next find the vertical distance on the ground glass between those points, call it H. (The transparent six inch metric ruler I found at Office Depot comes in handy for that.) Let R be the pivot radius, i.e., the distance from top of the front standard to the axis it pivots on, wherever that is. Then according to Wheeler, you should move the top of the front standard x units forward where

$$x = \frac{W}{H} R.$$

Let's do an example. I was trying to focus on a table top. The distance along the rail between the focus positions for the front of the table and the back of the table was about 6 mm. The vertical distance between those points on the ground glass was about 40 mm. The pivot radius was about 150 mm. So I had to move the top of the front standard $x = \frac{6}{40}150 = 22.5$ mm forward. One convenient way to move the front standard is as follows. You can get

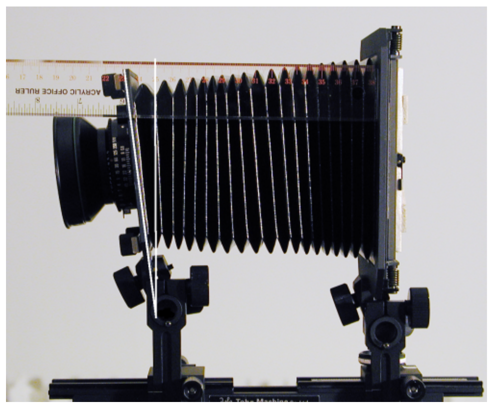

Figure 4: Tilting the lens plane

a small T-square with a metric scale, or just use a metric ruler which is wide enough to place square against the rear standard. Holding it there, measure the distance to the front standard and then move its top the additional distance x along the ruler. See Figure 4.

Needless to say, Wheeler's method and related procedures should just be taken as a first approximation. You should carefully examine the results on the ground glass, and correct as needed. For example, you could start by focusing on the far point. If you got the tilt right, the near point should also be in focus. But it may turn out that the focus has shifted. Consider a plane parallel to the film plane passing through the near focus point. The point of best focus in that plane may be *above* the desired near focus point, (so its image on the ground glass will be *below* where it should be.) See Figure 5. To bring it down (up on the ground glass), move the top of the front standard *back* to *decrease* the tilt. Similarly, if the actual point of best focus were *below* where it should be (*above* on the ground glass), to bring it up (down on the ground glass) move the top of the front standard *forward* to *increase* the tilt. One can calculate how much adjustment is necessary in terms of the parameter H used in Wheeler's method, but it usually will be so small that a slight nudge in the proper direction will suffice.

Another example may clarify this. I was photographing a line of stumps in the foreground all roughly at the same vertical height, and I wanted as much of the background in focus as possible. I chose infinity as my far focus point, and the top of the nearest stump as my near focus point. The focus spread along the rail between them was about 4 mm, and the vertical distance on the ground glass was about 80 mm. The pivot radius was about 150

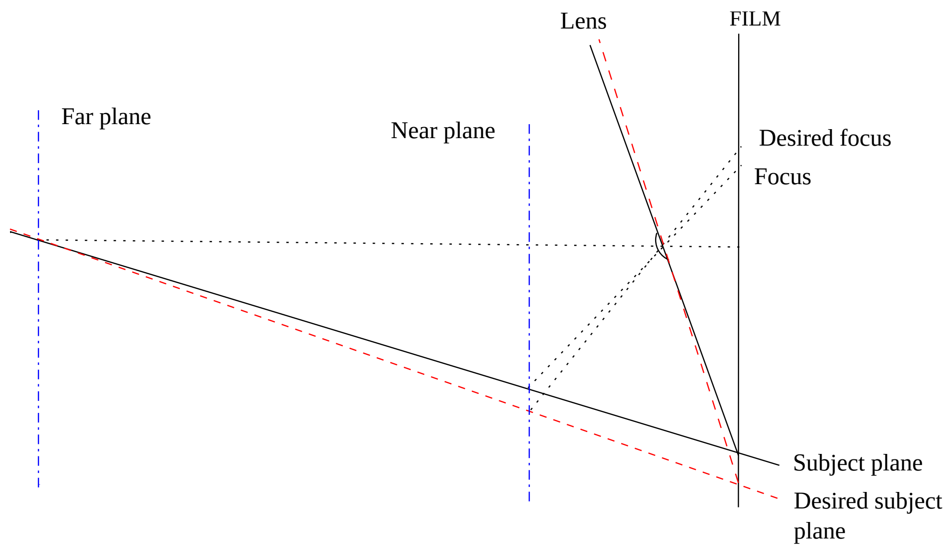

Figure 5: Error in placement of subject plane

mm. Wheeler's method suggested I move the front standard forward by $150(4/80) = 7.5$ mm. After refocusing on the far point (infinity), I found the actual focus point on the near stump was below the top of the stump (about 5 mm above it on the ground glass). That meant I had to increase the tilt slightly. Doing the calculation, which I won't go into here, I found that about a 6 percent increase in the quantity x would do it, but that amounted to about 0.5 mm. Just nudging the top of the front standard forward sufficed.

Note that if the actual focus in the near plane had been above the top of the stump (below it on the ground glass), I wouldn't have been able to see it since there wouldn't have been anything there. Focusing on the near point and looking at what happened to the focus at distant points also wouldn't help because that focus would have been below ground level and not visible. But there is another rule of thumb which could have been used in this case. Namely, suppose, as above, you first focus on the far point. If you need to bring the rear standard *back* (i.e., increase the film to lens distance) in order to bring the near point into focus, you need to *increase* the tilt. If you need to bring the rear standard *forward* in order to bring the near point into focus, you need to *decrease* the tilt.

Another way to check the tilt angle is to use a method advocated by Merklinger. (...) Imagine a plane through the lens parallel to the lens plane. This intersects the subject plane in a line, called the *hinge* line, and the distance from the lens to the hinge line is usually denoted J. See Figure 6. Then the top of the lens should be brought forward by

$$x' = R\frac{f}{J}$$

If the camera is level, you can visualize J by imagining a plumb line dropped from the lens and estimating where it intersects the subject plane. x' calculated this way shouldn't differ substantially from x calculated by Wheeler"s method. If it does, you can use a tilt intermediate between the two and check again on the ground glass.

One relatively common situation is that in which the subject plane is perpendicular to the film plane. That would be the case in many scenic photographs and in some table top photography. In that case, you often can measure J accurately with a metric tape.

For example, suppose one is using a 150 mm lens, and the lens is 1.5 meters = 1500 mm above the (flat) ground. Assume the pivot radius is 150 mm. Then one would move the top of the front standard $x = 150\frac{150}{1500} = 15$ mm forward.

These methods can be used with swings, but you use the horizontal distance on the ground glass, and you swing one side of front standard the requisite distance instead of its top. If you use both swings and tilts, it is not feasible to use these methods. In effect, the lens plane is pivoted about a skew axis in the lens plane and parallel to the film plane, but there is no simple way to determine the position of that axis.

If the region of interest in the subject plane is close to the lens, one must modify Wheeler's formula as follows. Consider the point P where the *untilted* lens axis meets the subject plane. Let u' be the subject distance to that point and v' the corresponding lens to film distance when the lens is focused on that point with the lens plane and film plane parallel. Then one must divide the previous value by

$$\frac{v'}{f} = \frac{u'}{u' - f} = 1 + M'$$

where M' is the untilted scale of reproduction for points at distance u'. In other words,

$$x = R\frac{W}{H}\,\frac{1}{1 + M'}.$$

After the lens is tilted, the distance from the lens to the film plane will increase to a larger value v, and the distance from the lens to P may also change, depending on the location of the tilt axis. In any case, if u' is sufficiently large $\frac{u'}{u'-f}$ is approximately 1, and we can ignore this factor, as above.

One can estimate $\frac{v'}{f} = 1 + M'$ by measuring v' directly and dividing by f, or by measuring the extension beyond the infinity focus, dividing that by f and adding one.

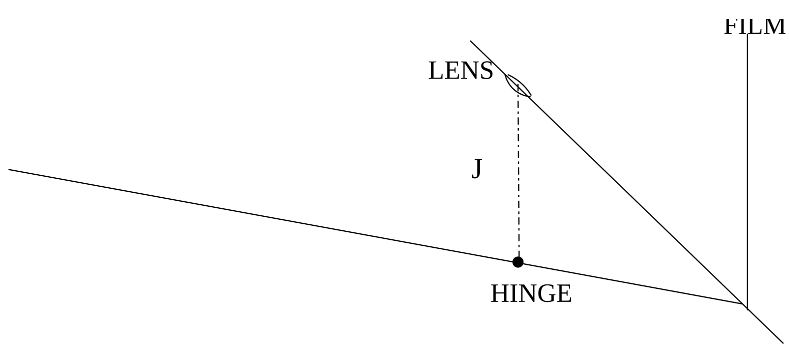

Figure 6: The hinge line

5.2 Getting adequate depth of field when lens is tilted

The previous discussion applies if you know in advance exactly where you want to put the plane of exact focus. Often, however, you want an entire region to be adequately in focus. Determining just where to focus and which f-stop to use to maximize the region of adequate focus is more complicated when the subject plane is tilted. If you fix the focal length, the f-stop, and the tilt angle, the region in space that corresponds to the depth of field in the untilted case is a wedge emanating from the hinge line. Let's call it the DOF wedge. See Figure 7. The DOF wedge is more or less centered on the plane of exact focus in typical

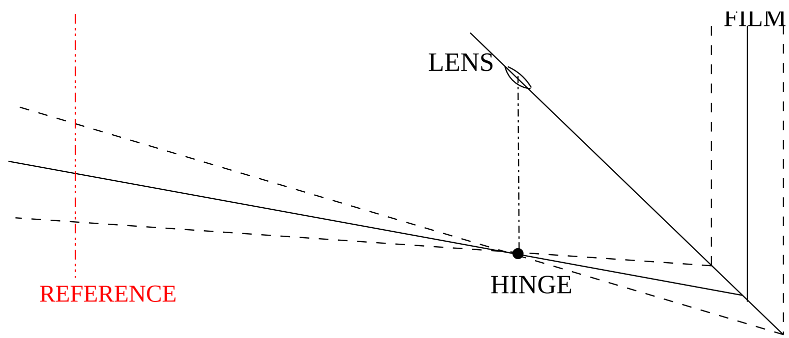

Figure 7: Depth of field wedge for tilted lens

situations. More precisely, consider a reference plane parallel to the film plane. If the back is vertical, that would mean a vertical plane at some fixed distance from the lens. Consider the intersection of the DOF wedge with the reference plane. It will be bounded above and below by the lines where the near DOF plane and far DOF plane meet it. The line where the principal plane of focus meets it will be halfway in between. Call that equal distance the split distance. Then, as Merklinger and Wheeler point out, for a reference plane at the hyperfocal distance, the split distance is $J\frac{f}{v}$, where J is the distance from the lens to the hinge line, f is the focal length, and v is the perpendicular distance from the lens to the film plane. In most cases $\frac{f}{v}$ is just slightly less than 1, so that distance is approximately J. At any other distance u from the lens, split distance will be $\frac{J}{H}\frac{f}{v}u$ or approximately $\frac{J}{H}u$.

Let me note in passing that the above description of the depth of field region is not quite accurate. When the lens plane is tilted with respect to the film plane, the circle of confusion becomes an ellipse in the film plane, and its dimensions vary with the tilt angle and position in the film plane. The result is that the boundaries of the depth of field region are not actually planes. Wheeler has estimated the departure from a plane for points on the midline of the image. I recently extended this to arbitrary points in the film plane. It turns out that for typical lens tilts, the discrepancy is relatively small. But for very large tilts and close subjects it could be noticeable. We shall ignore this error in what follows since it seldom has any practical significance.

Consider next what happens as you focus by moving the rear standard along the rail. The entire wedge pivots on the hinge line, and the angle it subtends at the hinge changes. But if you look at the trace of the DOF wedge on any fixed reference plane, its width stays the same with the exact focus splitting it in two.

So how should you use these facts in practice when you want to get as much as possible in focus? First, you should not expect miracles. Always see if you can obtain what you want with the lens plane parallel to the film plane just using depth of field at an appropriate f-stop. If that doesn't work, it will be because you have something in the foreground which you can't keep in focus without sacrificing the background, which usually means some part of the subject is well short of the hyperfocal distance. By tilting the lens, you will be able to do a better job of keeping parts of the foreground and background in focus, but you will pay a price in depth of field transverse to the exact plane of focus at any given f-stop. This may be particularly limiting for elements of the subject in the foreground.

So consider a typical problem in landscape photography, some flowers fairly close to the lens and something such as trees or hills with vertical extent in the background. To be definite, let's suppose you have a hill at some distance from the lens. If the flowers also have vertical significant extent, tilting the lens is not going to do much good, so suppose that is not the case.

Let me describe two methods you might use.

First method

Keeping in mind the vertical split of the wedge, pick a point on the hill about half way up as your distant point and a point in the foreground as your near point. Use those to determine your tilt, and note the position of the rear standard on the rail. Next move the rear standard back and forth and note the locations on the rail where the plane of exact focus passes through the top and bottom of the hill. Make sure those planes encompass all the interesting detail at other distances also. In particular, if you have a problem in the foreground, that suggests you didn't get the tilt angle right, so try adjusting it slightly as indicated in 5.1.

You now have determined a *focus spread* on the rail just as previously. The position of best focus should be approximately in the middle. (It should actually be slightly closer to the lens than that, but in common situations not enough to bother. See Section 7.) It should also be quite close to where you set it when fixing the tilt. If you could focus exactly, there would be no error, but that is never possible in the real world, so you should tolerate a small error. If it is larger, you may want to again try adjusting the tilt slightly to improve things.

Second method

Pick as your far point the *highest* point on the hill and a near point as before. Use these to determine the tilt angle. (It should be less than that produced by the first method.) Note the position on the rail. Move the rear standard so the lowest desired point on the hill comes into focus and note that position on the rail. Use these two points to establish the focus spread instead, and move the standard to its center to establish the plane of exact focus.

The second method may have the advantage of yielding a smaller focus spread and hence it may allow for a larger aperture. To see why, look at the expression for the split distance at distance u from the lens. It is

$$\frac{J}{H}\frac{f}{v}u = \cdots = \frac{Nc/f}{\sin(\phi)v/f}u.$$

where the dots represent some algebra that I omit. On the other hand, if the tilt angle is not too large and the subject plane not too steeply inclined with respect to the horizontal, it turns out that v/f is pretty close to one. That means that as you decrease ϕ (hence $\sin(\phi)$), if you compensate by decreasing N, you can keep the split distance at any given distance from the lens more or less fixed. Indeed you can match any change in the amount you move the top of the front standard forward by a proportional change in the f-number.

One problem with the second method, however, is that you are relying on the calculations to work to be sure important parts of the foreground remain in focus. With the first method, you can see that they are in focus with the lens wide open.

Having established the tilt one way or another, you want to determine which f-stop to use to make sure the wedge is broad enough to encompass everything of interest. You can use essentially the same formulas as before to do that in terms of the focus spread. But the basic formula for the rail distance on either side of the midpoint focus position becomes

$$d = (Nc)\,Q \text{ with } Q = \frac{v}{f},$$

where v is the distance from the lens to the film plane and f is the focal length. To see how this is related to the previous formula, note that

$$Q = \frac{v}{f} = \frac{1+M}{\cos(\phi)}$$

where ϕ is the tilt angle, and M is the scale of reproduction, after tilting, for the point P where the untilted lens axis intersects the subject plane.

Let w be the focus determined as above. Then, the formula for N becomes

$$N = \frac{w}{2c}\frac{1}{Q}.$$

For typical scenes, Q is not much larger than 1, and, in any event, if you ignore it, all you do is overestimate the needed f-number, which usually does no harm. So in practice it is usually safe to use the same formula as before

$$N = \frac{w}{2c}.$$

If you want a more precise estimate, you need to determine the factor $Q = \frac{v}{f}$. This is a little tricky because v is the perpendicular distance from the rear nodal point of the lens to the film plane. The rear nodal point is usually close to the center of the lens, but for telephoto or reverse telephoto lenses, it may be elsewhere, possibly not even in the lens. It is possible to determine its location, if you know the focal length, and of course you only need to do it once for each lens. But one way to estimate v without knowing the position of the node is as follows. With the lens tilted, find where a line perpendicular to the film plane through the lens intersects the film plane. Looking at the ground glass, determine as best you can the position on the rail where infinity comes into focus at that point. Then refocus on the desired subject plane, and measure the distance beyond that infinity focus. Finally, to determine v add the displacement to $f/\cos(\phi)$ where ϕ is the tilt angle.

6 Comparison with smaller formats

It is interesting to ask how large format photography differs from medium and 35 mm photography in applying these principles. I went back and looked at my Horseman, and it was clear why they aren't too useful for it. For medium format, one uses a smaller value of c, and also one tends to use wider apertures. So the distance $d = \mathrm{Nc}$ is smaller on both counts. There was no advantage to putting a focusing scale on the Horseman focusing knob. Indeed it was more accurate just to measure the travel along the bed.

But after some further thought I realized that some of the same principles are used in most medium and 35 mm format lenses. Namely, lenses used for such cameras usually have depth of field scales. The f-numbers marked in pairs on the lens barrel are just indicators of displacement along the lens axis from the exact focus position. As noted above, displacements of the lens along its axis are very small, but they are greatly magnified by gearing for for purposes of focusing. The manufacturer decides what the parameter c should be, and that determines how the f-numbers are placed on the lens lens barrel. But you could use these markings in a manner similar to that discussed previously. If you think of the f-numbers as an expanded distance scale using nonstandard units, it amounts to the same thing.

7 Exact formulas and when they are needed

Let me state the exact formulas for the distance d within which image points produce acceptable blurs in the film plane. I will concentrate on the case where the lens and film planes are parallel, but similar considerations apply in the case of tilted lens plane.

It turns out that the distances on either side of the film plane are not quite the same. Let v_1 denote the image distance for the nearest point of interest, v_2 the image distance for the furthest point of interest, and v the distance for the film plane. Then

$$v_1 - v = \frac{Nc\frac{v}{f}}{1 - \frac{Nc}{f}} \tag{2}$$

is the distance on the side of the film plane away from the lens, and

$$v - v_2 = \frac{Nc\frac{v}{f}}{1 + \frac{Nc}{f}} \tag{3}$$

is the distance on the side of the film plane closer to the lens. The total distance between the near and far focus points is

$$v_1 - v_2 = \frac{2Nc\frac{v}{f}}{1 - (\frac{Nc}{f})^2} \tag{4}$$

As before, $\dfrac{v}{f} = 1 + M$ where M is the scale of reproduction for image points in the film plane (or the same thing divided by the cosine of the tilt angle for tilted lens plane.)

Usually Nc is quite small compared to f. For example, if $c = 0.1$ mm, $N = 45$, and $f = 100$ mm, $\frac{Nc}{f} = \frac{4.5}{100} = 4.5$ percent. The smaller f gets, the larger $\frac{Nc}{f}$ becomes. But for most practical large format photography, we may ignore those terms in the denominator, and we obtain

$$d = Nc\frac{v}{f} = Nc(1+M) = N_e c$$

as before. But there are cases where the error made by using the approximate formulas, though small, can still have a noticeable effect, and you can do better using the exact formulas.

If one does the algebra to solve the above two equations simultaneously, one obtains

$$v = \frac{2v_2 v_2}{v_1 + v_2} \tag{5}$$

$$\frac{Nc}{f} = \frac{v_1 - v_2}{v_1 + v_2}. \tag{6}$$

Denote the second of these quantities by e. As before, let $w = v_1 - v_2$ be the focus spread between near and far focus points. Assume that you have measured v_2 and w. Then you can calculate $v_1 = v_2 + w$ and

$$e = \frac{v_1 - v_2}{v_1 + v_2} = \frac{w}{v_1 + v_2}.$$

You can then use e to determine the proper placement of the film plane. Namely, it should be

$$d_2 = \frac{w}{2}(1-e) \tag{7}$$

from the far focus point and

$$d_1 = \frac{w}{2}(1+e) \tag{8}$$

from the near focus point. Thus, you should favor the far focus point by the amount e, relative to what you would do if you chose the midpoint.

You can also calculate N by putting the above expression for e in $\frac{Nc}{f} = e$ and solving for N.

$$N = \frac{f}{c}\frac{w}{v_1 + v_2} = \frac{w}{2c}\frac{f}{\frac{v_1+v_2}{2}}.$$

But $v_0 = \frac{v_1+v_2}{2}$ is just the position of the midpoint between the focus points and $\frac{f}{v_0} = \frac{1}{1+M_0}$ where M_0 is the scale of reproduction at the midpoint. *So this gives exactly the same result for the f-number as that we stated before*, provided we take the magnification into account.

It should be noted as previously that measuring distances to the lens plane is a little tricky. Either one must have located the rear principal point precisely, or one can focus carefully on infinity, measure the extension beyond that point, and add that to the focal length. (If the lens plane is tilted, you need to divide the focal length by the tilt angle and add that.)

How often is the quantity e, which gives the percentage shift, significant? Solve $e = \frac{Nc}{f}$ for

$$f = \frac{Nc}{e},$$

and take $c = 0.1$ mm, $N = 64$. Then $f = \frac{6.4}{e}$. For $e = .1 = 10$ %, this says $f = 64$ mm; for $e = .2 = 20$ %, it says $f = 32$ mm, and for $e = .5 = 50$ %, it says $f = 12.8$ mm. Thus, this only begins to be significant for extreme wide angle lenses, and for those, only for very small apertures. But how often would one be using such small apertures with a wide angle lens? Consider for example, a 50 mm, lens. At f/64, with $c = 0.1$ mm, $e = \frac{6.4}{50} = .128 = 12.8$ %. Also the hyperfocal distance is $\frac{2500}{6.4}$ or about .39 meters. This is less than ten times the focal length, which is usually considered within close-up range. So, even for extreme wide angle lenses, this is unlikely to be an issue except for close-ups, since for such lenses, very small apertures are not generally needed to obtain adequate depth of field.

When the lens plane is tilted with respect to the film plane, the errors we have been discussing are larger. The distances on either side of the film plane corresponding to the boundaries of the depth of field wedge are multiplied by a factor $\frac{v}{f}$ which is larger than above because of the tilt. But the algebra is more complex, and I haven't figured out a good way to estimate the correction. From some simulations, it appears that even for a 90 mm lens (4 x 5 format), one should favor the position closer to the lens slightly rather than centering the standard exactly in the middle.

8 Some brief remarks on focusing error

A refined distance scale can also help you understand focusing errors. Assume that the film plane, lens plane, and subject plane are parallel. Suppose also that the lens is wide open, so that sets the f-number N. If you fix one subject plane and try to focus on it, the standard formula governing what is called depth of focus is

$$2d = 2\frac{v}{f}Nc = 2(1 + M)Nc \text{ or just } 2Nc \text{ for distant subjects.}$$

This is the same formula we used previously, but N is the f-number for the lens wide open, and the interpretation of the variables d and c, is different. d gives you the distance along the rail on either side of the theoretically correct focus position such that the subject just comes into focus or just goes out of focus. If you focus repeatedly on the same subject plane, the total range over which you may vary is $2d$.

c is also the diameter of a circle of confusion, but in this case it is really a measure of how closely you can distinguish separate points on the ground glass. It may not be the same as the c used previously. One way to think of this is that what you see on the ground glass is comparable to what you would see in a 4 x 5 transparency if you took the picture with the lens wide open. At least that would be true if you viewed both ground glass and the transparency from the same distance, and there is no appreciable difference in the position of the ground glass and the film. But, just as for prints, the viewing distance is absolutely critical. If you use a loupe, either to look at the ground glass or the transparency, what you are doing is getting your eye closer to what you are looking at. All the optics of the loupe does is allow you to focus on it. Of course, the closer you get, the more detail you will see. Finally, generally you would expect the film to resolve more detail than the ground glass, so under high magnification you would expect to see more in the transparency than on the ground glass.

With these thoughts in mind, I did some experiments to check how consistently I could focus. There are three ways I focus. I have a pair of reading glasses which allows me to place my eye 6 - 7 inches from the ground glass. That would be roughly the same as looking at the ground glass with about 2 x magnification. I also have a 3.6 X and 7 X loupes. I find the typical range from just in to just out for an f/5.6 lens are as follows

Mag	$2d$	corresponding c
2 X	less than 1.5 mm	0.134 mm
3.6 X	less than 0.7 mm	0.063 mm
7 X	less than 0.3 mm	0.027 mm

These values are generally consistent with the corresponding values often chosen for the circle of confusion, but with my relatively coarse ground glass, a bit higher than what I would use for depth of field calculations, e.g, consider my choice of 0.1 for 4 x 5 format, which corresponds to a coc of 0.2 in an 8 x 10 print viewed at about 12 inches.

Note that looking at the ground glass at different magnifications corresponds to viewing different size prints at about 12 inches.

Mag	Print size
2 X	8 x 10
4 X	16 x 20
8 X	32 x 40

Of course, viewing the prints further away changes things. In principle a 16 x 20 print viewed at 24 inches should look like an 8 x 10 print viewed at 12 inches, at least as far as perspective and visual resolution are concerned.

For close-ups, one must multiply by the factor $1 + M$. In the extreme case of scale 1:1, this factor would be $1 + M = 1 + 1 = 2$, so the values in the above table would all have to be doubled. So even with a high power loupe, there would be a considerable range, as large as half a mm, in which there would be no clearly best position.

There are several not very startling conclusions one can draw from this discussion. First if you shoot wide open, you may be able to see your focusing error in an 8 x 10 print, so that is a bad idea. Second, if you use a 4 X loupe, you may still be able to detect focusing error if you get very close to a large print, but if you stand back to view such a print at a more reasonable distance, focusing error should not be a significant factor. Third, it is not often the case that focusing with a 7 X or higher magnification loupe will lead to detectable differences in a print.

Most important, these considerations tell you that a certain amount of focusing error is *inevitable*. Optical theory tells us there is a range where you can't distinguish better focus from worse focus, which depends on the aperture you focus at and the degree of magnification you use when focusing. You can reduce it by using a loupe and a finer, brighter ground glass. You can also try to make it work for you rather than against you by systematizing how you focus. For example, in using the midpoint method, you might always approach the near point from further back from the lens and the far point from closer to the lens. Then your tendencies to overshoot might cancel out when determining the midpoint. In addition, if you have very fine control of position of the standard on the rail, you can make repeated attempts at focusing and note the positions. If there is no systematic bias, and your choices are randomly chosen within the small focusing range, then probability theory tells you that averaging them to choose a final position will yield a more accurate value. But no matter what you do, you will never eliminate it entirely.

Finally, in relation to our previous discussions, we see that focusing error will play an important role in estimating the near and far focus points, particularly if they are close together. That underlines the previous caveat that the methods discussed in this article should only be considered first approximations and *must always be adjusted on the basis of what you see on the ground glass*.