An infinite descent into pure mathematics

by Clive Newstead
1. What I learnt about learning
2. Why I wrote a textbook
3. Developing the book
4. Preview of the book
5. Concluding remarks
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Traditional mathematics class

**Course design**
- Choose topics to be covered
- Choose grade weightings for assessments
- Choose textbook & set schedule

**Classes**
- Lecturer presents content
- Maybe some question-and-answer interaction

**Assessment**
- Problem sheets
- Quizzes and examinations
Current research about teaching and learning

Good practices to maximise student learning:

- Proper alignment of learning objectives, teaching strategies and methods of assessment
- Engaging students actively in the learning process
- Use of inquiry-based strategies and assessments
- Using a variety of teaching methods and assessments

Alternative teaching model

**Course design**
- Choose learning objectives
- Design classes around helping students achieve LOs
- Choose types of assessment that test these LOs

**Classes**
- Before: some content delivery (e.g. reading, video)
- During: activities targeting LOs, clarifications, some lecture

**Assessment**
- Problem sheets, quizzes, examinations
- Projects, presentations, group work, ...
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Concepts of Mathematics — description

From course catalogue:

“This course introduces the basic concepts, ideas and tools involved in doing mathematics. As such, its main focus is on presenting informal logic, and the methods of mathematical proof.  […list of topics…]”

From department website:

Concepts of Mathematics — course design

Learning objectives

- “Presenting informal logic” = communication
- “Methods of mathematical proof” = problem-solving

Mathematical topics

- Symbolic logic, sets, functions
- Induction on the natural numbers
- Number theory
- Combinatorics
- Other topics (real numbers, probability theory, basic topology, ...)

Syllabus: ✓ Next step: find a textbook.
Concepts of Mathematics — textbook

**Textbook criteria:** A textbook for Concepts should...

- Be of an appropriate length
- Cover enough mathematical topics
- Cover communication and problem-solving skills
- Practise what it preaches
- Be as agnostic as possible
An infinite descent into pure mathematics
Concepts of Mathematics — textbook

Solution: Write my own notes

Time frame: 51 days

Backup plan: Concede
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Design considerations

Lots of decisions to make

- Mathematical areas to cover
- Definitions and theorems to emphasise
- Include exercise solutions or not?
- Level of difficulty
- Choices of convention
- Level of verbosity
- Level of detail in proofs
- General tone of the book
- How to cover both skills and content
- Name of the book
- Licensing and copyright issues
Example dilemma #1

What is a function?

Possible definitions:

(1) A set of ordered pairs such that . . .
(2) A triple \((X, Y, f)\) where \(f \subseteq X \times Y\) such that . . .
(3) A rule assigning to each \(x\) a unique \(y\)
(4) An imaginary machine taking inputs and giving outputs
(5) A primitive notion in terms of which all other mathematical notions are defined

My choice: (3) because it is the most agnostic
Example dilemma #2

Is zero a natural number?

Possible resolutions:

(1) Yes
(2) No
(3) Choose your own adventure, make explicit when needed
(4) Use $\mathbb{N}_0$ and $\mathbb{N}_1$ (or similar)

My choice: (1) for lots of reasons
Example dilemma #3

What is the best domain of discourse for number theory?

Possible answers:

(1) $\mathbb{Z}$
(2) $\mathbb{N}$ (with or without zero?)
(3) Sometimes $\mathbb{N}$, sometimes $\mathbb{Z}$

My choice: (1) because it generalises easily to more general rings
Return to learning objectives

**Communication skills**
- Using notation accurately
- Developing mathematical fluency
- Evaluating effectiveness of others’ proofs
- Typesetting in \LaTeX

**Design principles**
- Write accurately and clearly
- Include discussion exercises
- Provide guidance on how to structure a proof
- Provide \LaTeX support
Return to learning objectives

**Problem-solving skills**
- Identifying feasible proof strategies
- Identifying relevant definitions and theorems
- Creativity in problem-solving approaches

**Design principles**
- Examples and exercises galore
- Vary level of difficulty
- Provide problem-solving tips
- Do not provide solutions to exercises
And so the writing began

*Note: this is not actually me... I don't have a Mac*
## Growth of the book

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
<th># pages</th>
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<td>May 9, 2015</td>
<td>Started writing</td>
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<tr>
<td>Jun 29, 2015</td>
<td>Started teaching 21-127</td>
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Next steps

Remaining tasks

- Finish remaining chapters
- Add more examples, discussions & exercises
- Add more guidance for communication and proof-writing
- Add more diagrams and graphics
- Include chapter introductions, reflections and summaries
Reflection

What I have learnt

- Writing a textbook takes a lot of time and effort
- Writing a textbook does not contribute towards PhD requirements
- You can’t make everyone happy
- \LaTeX\ is full of surprises
- Having a project to work on is fun
- This book might never be finished
Thanks for listening!

Website
www.infinite-descent.xyz

These slides
math.cmu.edu/~cnewstea/talks/20180301.pdf