

15-151 Homework 5

Please submit in class at 8:00am on Thursday 20th July

Exercises

1. In Question 1, sets A , B , C and D are defined by:

$$A = \{n \in \mathbb{N} \mid n \leq 64\}, \quad B = \{n \in A \mid n \text{ is divisible by } 7\},$$

$$C = \{n \in A \mid \exists k \in \mathbb{N}, n = 2^k - 1\}, \quad D = \{n \in A \mid \exists k \in \mathbb{N}, n = k^3\}$$

Write out the following sets in list notation:

(a) $B \cap C$; [2 points]

(b) $B \cap D$; [2 points]

(c) $C \cup D$; [2 points]

(d) $(B \setminus C) \cup (C \setminus B)$; [3 points]

(e) $(B \cup C) \setminus (B \cap C)$. [3 points]

2. Let X and Y be sets.

(a) Prove that $(X \setminus Y) \cup (Y \setminus X) = (X \cup Y) \setminus (X \cap Y)$ [7 points]

(b) Prove that $X \subseteq Y$ if and only if $Y \setminus (Y \setminus X) = X$. [7 points]

3. Let U and V be sets. Must it be the true that $\mathcal{P}(U \cup V) = \mathcal{P}(U) \cup \mathcal{P}(V)$? Prove your claim. [9 points]

Optional but recommended tasks (not for credit)

4. Suppose there is a set of all sets—call this set \mathcal{S} . Define a set B by

$$B = \{X \in \mathcal{S} \mid X \notin X\}$$

Prove that $B \in B$ if and only if $B \notin B$. This is nonsense. What went wrong?